

Plasma Physics and Technology

Basic concepts

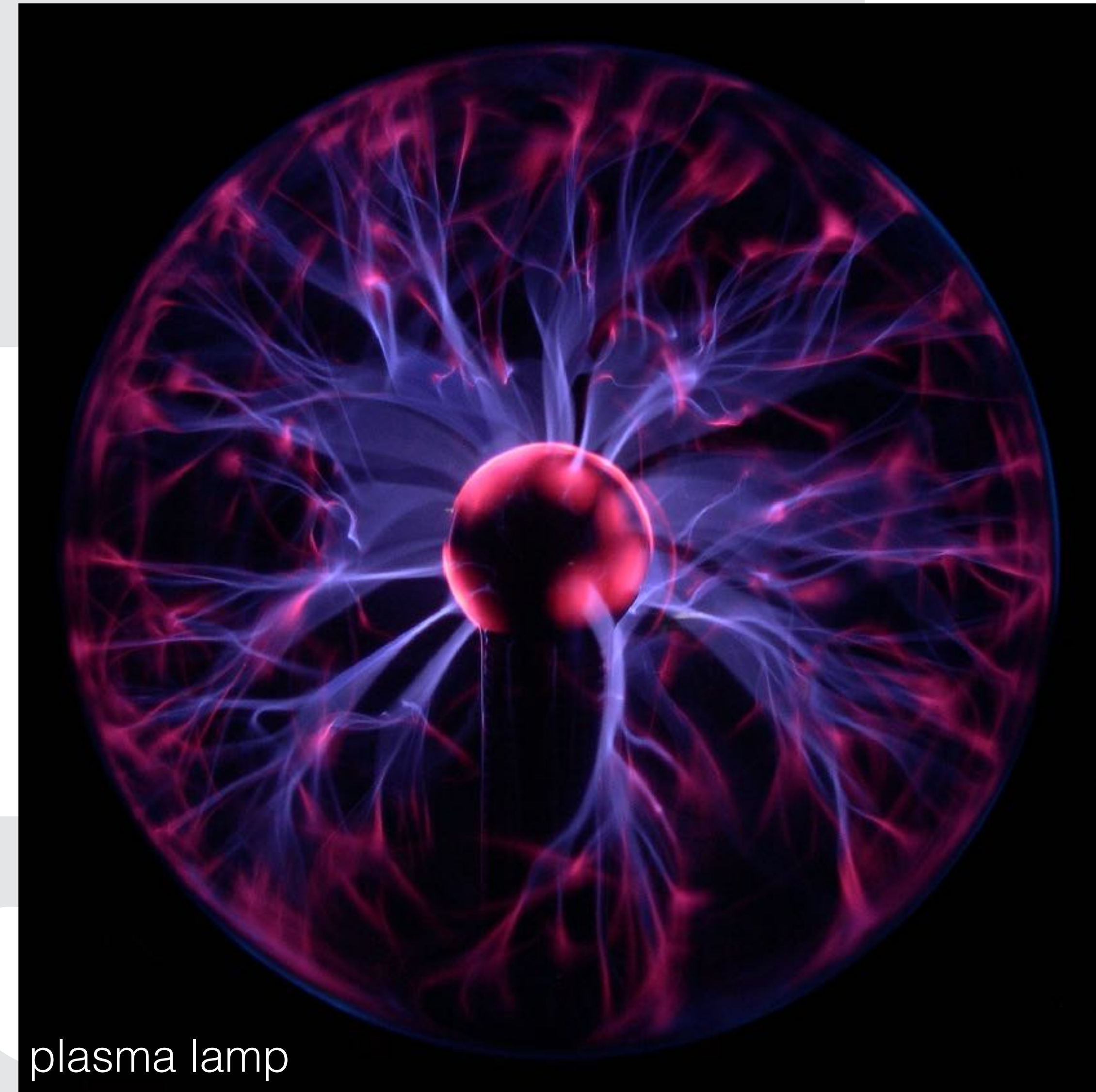


Mestrado Integrado em Engenharia Física
Tecnológica

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Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico

- general info
- what is a plasma
 - definition
 - examples
 - applications
- basic concepts
 - Debye shielding
 - plasma parameter
 - plasma frequency

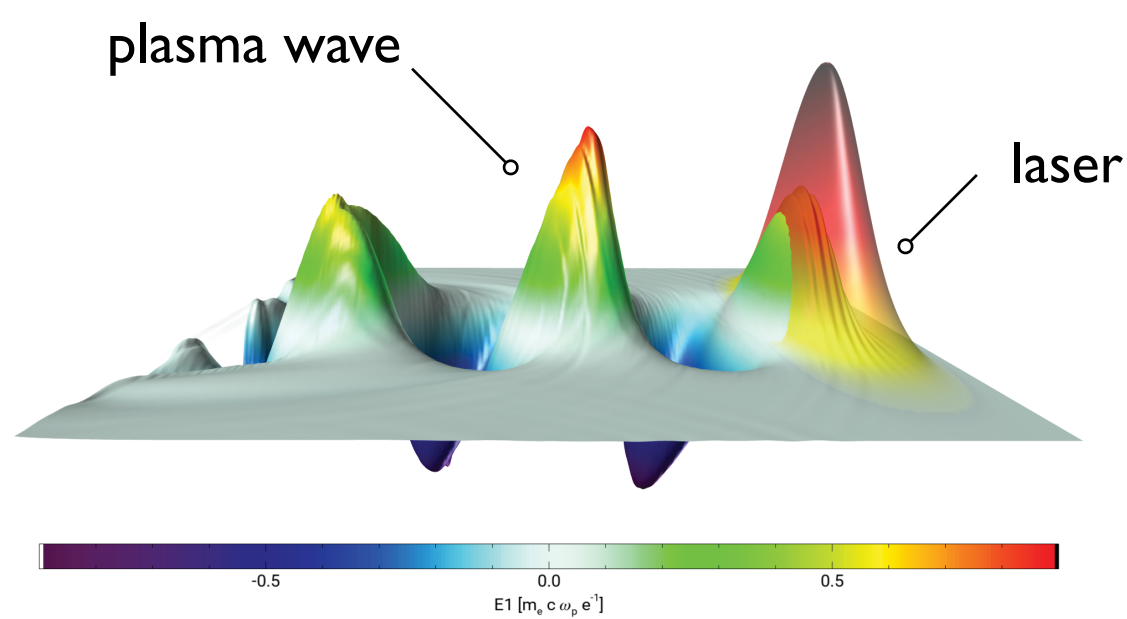


- we will have a few invited speakers to talk about their plasma research at IST in a few classes
- mid-term test (tentatively on 11 Nov 2019) + 2nd test on regular exam period or exam
- reference textbooks
 - Physics 222 ABC, Introduction to Plasma Physics, John Dawson, Academic Publishing Service (UCLA)
 - Introduction to Plasma Theory and Controlled Fusion, Francis Chen
 - Introduction to Plasma theory, Dwight R. Nicholson

- lecturers:
 - Prof. Jorge Vieira (theoretical lectures) - ext. 3375
 - Prof. Hugo Terças (problem classes, exercices can be found online in the course website)
- office hours: 10.30-12.30 am on Mondays.
- general guidelines for problem classes:
 - solve problems before class.
 - you will have the opportunity to work on some problems independently during class.
- there will be computational examples to illustrate some of the concepts in class
 - computational examples can be ran online by accessing our server using your internet browser or by downloading the code from github.
 - Server address: <https://zpic.tecnico.ulisboa.pt/hub/login>
 - accounts/passwords + instructions for simulation server to be distributed soon

computational tool

- **Particle-in-cell Code suite.** Fully relativistic electro-magnetic 1D and 2D (FDTD/spectral) and 1D Electrostatic.
- **Python interface.** All simulation codes/parameters can be controlled via Python interface.
- **Educational examples.** Set of Python Jupyter notebooks with detailed physics problem description and simulation setup.



Come find us on GitHub
github.com/zambzamb/zpic

compact plasma accelerator



Example notebook

Theoretical introduction

Electron Plasma Waves

Created by Rui Calado and Jorge Vieira, 2018

In this notebook, we are going to study the dispersion relation for electron plasma waves.

Theory

Electron plasma waves are longitudinal waves that may propagate in unmagnetized plasmas. To derive the dispersion relation for such waves let us start by considering the following setup:

- $\nabla \times \mathbf{E} = 0$ (longitudinal waves)
- $T_i = T_e = 0$ (Cold plasma)
- $\mathbf{B} = 0$ (Unmagnetized)

We start by writing the continuity and momentum equations for the electron and ion species:

$$\begin{cases} \frac{\partial n_{e,i}}{\partial t} + \nabla \cdot (n_{e,i} \mathbf{v}_{e,i}) = 0 \\ \frac{\partial \mathbf{v}_{e,i}}{\partial t} = \mp \frac{e}{m_{e,i}} \mathbf{E} \end{cases}$$

Then we consider Poisson's equation:

$$\epsilon_0 \nabla \cdot \mathbf{E} = e(n_i - n_e).$$

Applying a time derivative twice,

$$\epsilon_0 \nabla \cdot \left(\frac{\partial^2 \mathbf{E}}{\partial t^2} \right) = e \left(\frac{\partial^2 n_i}{\partial t^2} - \frac{\partial^2 n_e}{\partial t^2} \right).$$

Using the continuity and momentum equations we get:

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - \frac{e^2 n_0}{\epsilon_0} \left(\frac{1}{m_i} + \frac{1}{m_e} \right) \mathbf{E} = 0.$$

This is the equation for a harmonic oscillator. Neglecting the $1/m_i$ term since $m_e \ll m_i$, our oscillation frequency is the electron plasma frequency:

$$\omega = \frac{e^2 n_0}{\epsilon_0 m_e} \equiv \omega_p.$$

Warm plasma

Now, what happens if we consider a warm plasma instead? Neglecting ion motion, the first step is to add a pressure term to the electron momentum equation:

$$\frac{\partial \mathbf{v}_e}{\partial t} = -\frac{e}{m_e} \mathbf{E} - \frac{\gamma k_B T_e}{m_e n_0} \nabla n_e.$$

We also note that Poisson's equation now takes the form:

$$\nabla \cdot \mathbf{E} = -\frac{e}{\epsilon_0} n_i.$$

simulation initialisation

We also note that Poisson's equation now takes the form:

$$\nabla \cdot \mathbf{E} = -\frac{e}{\epsilon_0} n_i.$$

Taking the divergence of the momentum equation, we get:

$$\nabla \cdot \left(\frac{\partial \mathbf{v}}{\partial t} \right) = -\frac{e}{m_e} \nabla \cdot \mathbf{E} - \frac{\gamma k_B T_e}{m_e n_0} \nabla \cdot (\nabla n_e).$$

Using the unchanged continuity equation and Poisson's equation:

$$\frac{\partial^2 n_i}{\partial t^2} + \omega_p^2 n_i - \frac{\gamma k_B T_e}{m_e} \nabla \cdot (\nabla n_i) = 0.$$

Considering the high frequency regime, there will be no heat losses in our time scale, and so we will take the adiabatic coefficient $\gamma = 3$ for 1D longitudinal oscillations. Additionally, we use the definition $v_{th}^2 = k_B T_e / m_e$ to write:

$$\frac{\partial^2 n_i}{\partial t^2} + \omega_p^2 n_i - 3v_{th}^2 \nabla \cdot (\nabla n_i) = 0.$$

The final step consists of considering sinusoidal waves such that $n_i = n_1 \exp(i(kx - \omega t))$ and then Fourier analyzing the equation ($\nabla = ik$, $\frac{\partial}{\partial t} = -i\omega$), which results in the dispersion relation:

$$\omega^2 = \omega_p^2 + 3v_{th}^2 k^2.$$

Simulations with ZPIC

```
In [2]: import emids as zpzc

#v_the = 0.001
v_the = 0.02
#v_the = 0.20

electrons = zpzc.Species("electrons", -1.0, ppc = 64, uthe=[v_the, v_the, v_the])
sim = zpzc.Simulation(nx = 500, box = 50.0, dt = 0.0999/2, species = electrons)

sim.enf.solver_type = "PSATD"

sim.enf.set_filter("none")
#sim.enf.set_filter("sharp", a = 0.99)
#sim.enf.set_filter("gaussian", a = 0.001)

We run the simulation up to a fixed number of iterations, controlled by the variable 'niter', storing the value of the EM field E_i at every timestep so we can analyze them later:

In [3]: import numpy as np

niter = 4000
Ex_t = np.zeros((niter, sim.nx))
Ex_t = np.zeros((niter, sim.nx))

tmax = niter * sim.dt

print("\nRunning simulation up to t = {} s".format(tmax))
while sim.t <= tmax:
    print("a = {} s, t = {} s".format(sim.n, sim.t), end = '\r')
    Ex_t[sim.n, :] = sim.enf.Ex
    Ez_t[sim.n, :] = sim.enf.Ez
```

analysis and questions for discussion

Electromagnetic Plasma Waves

To analyze the dispersion relation of the electrostatic plasma waves we use a 2D (Fast) Fourier transform of $E_z(x, t)$ field values that we stored during the simulation. The plot below shows the obtained power spectrum alongside the theoretical prediction.

Since the dataset is not periodic along t we apply a windowing technique (Hanning) to the dataset to lower the background spectrum, and make the dispersion relation more visible.

```
In [4]: import matplotlib.pyplot as plt
import matplotlib.colors as colors

# (omega, k) power spectrum
win = np.hanning(niter)
for i in range(sim.nx):
    Ez_t[i, :] *= win

sp = np.abs(np.fft.fft2(Ez_t))**2
sp = np.fft.fftshift(sp)

k_max = np.pi / sim.dx
omega_max = np.pi / sim.dt

plt.imshow(sp, origin = 'lower', norm=colors.LogNorm(vmin = 1e-7, vmax = 0.01),
           extent = (-k_max, k_max, -omega_max, omega_max),
           aspect = 'auto', cmap = 'gray')

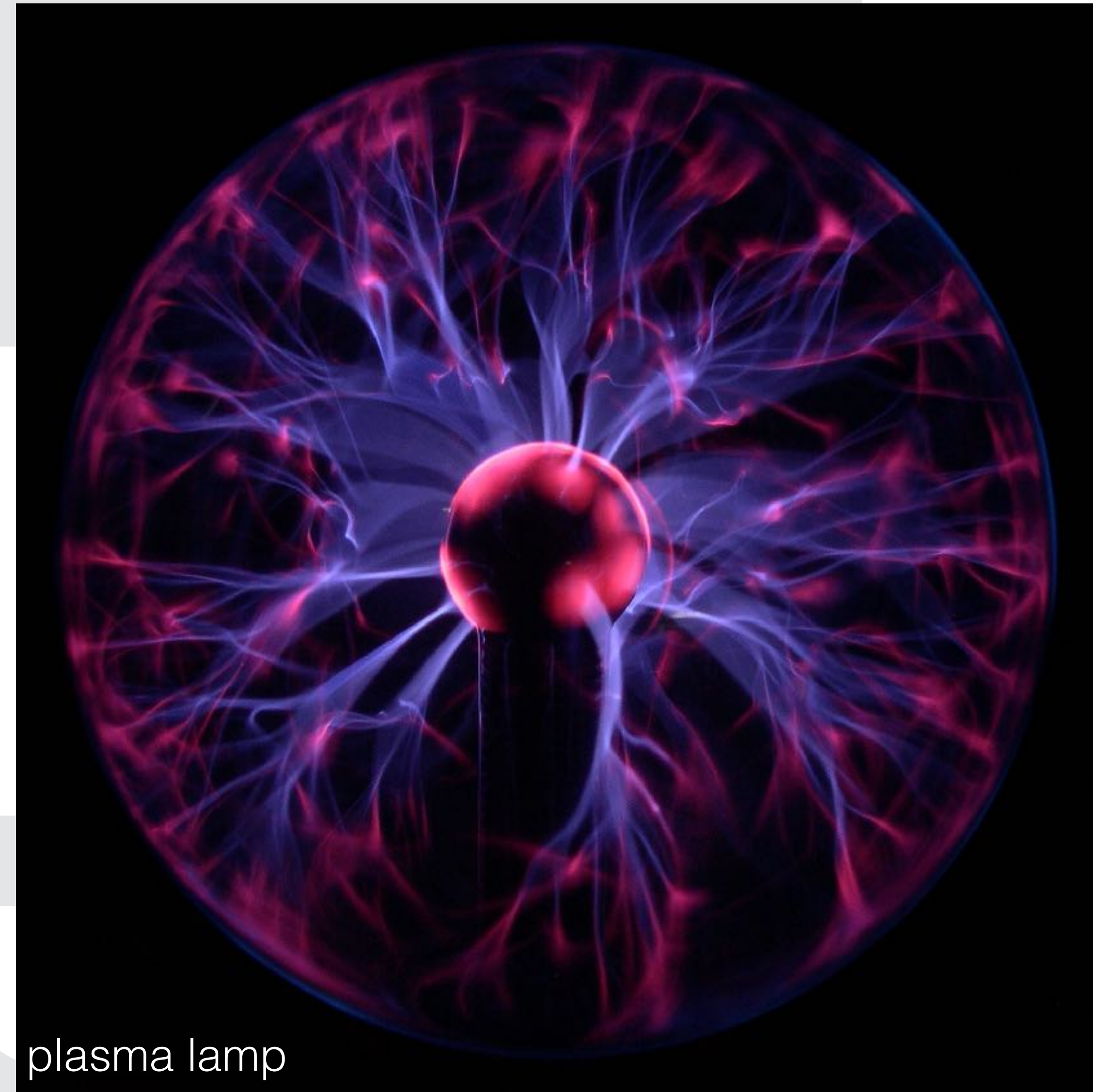
k = np.linspace(-k_max, k_max, num = 512)
wmp.sqrt(1 + k**2)
plt.plot(k, w, label = "\omega/\omega_p^2 = \omega_max/k^2 + k^2 c^2/28", color = 'r', ls = '--')

plt.ylim(0, k_max)
plt.xlim(0, omega_max)
plt.xlabel("k [1/m]")
plt.ylabel("\omega_max (s/omega_n^2)")
plt.title("EM-wave dispersion relation")
plt.legend()
plt.show()
```

EM-wave dispersion relation

- basic concepts (Debye shielding, plasma frequency, ...)
- single particle motion
- fluid description of the plasma
- introduction to waves in plasmas
- transport and diffusion in weakly and fully ionised plasmas
- introduction to kinetic theory
- Landau damping

- general info
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plasma lamp

- material with mobile charges (negative, positive, both) in which the electric and magnetic interactions dominate the dynamics of the system
- system can be globally neutral (most of our course is devoted to globally neutral plasmas), not neutral (we will discuss these also during the semester), or even consisting of a single type of particles
- long range nature of electromagnetic interactions imply collective motions in which particles move coherently
- these properties make the physics of plasmas extremely rich as we will see during the course.

Examples - hot gaseous systems

strongly heated gases such that some or all electrons detach from constituent atoms and molecules

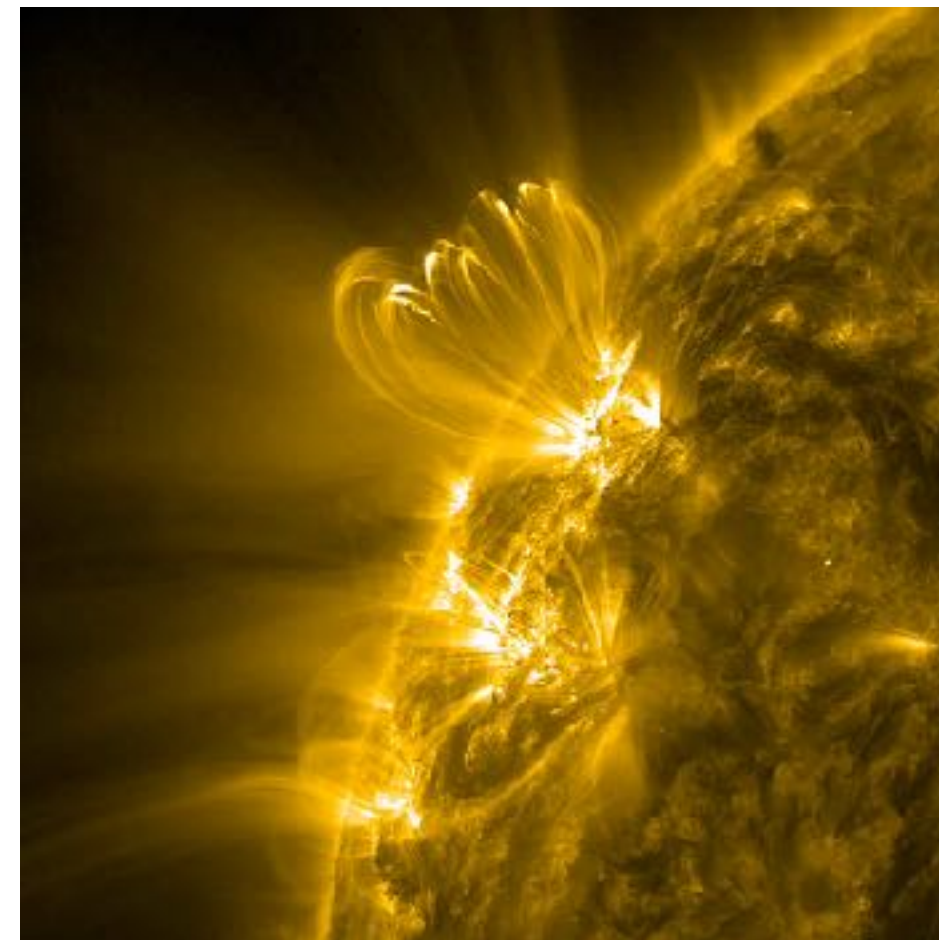
fusion reactor plasmas

tokamak



stars hot surface

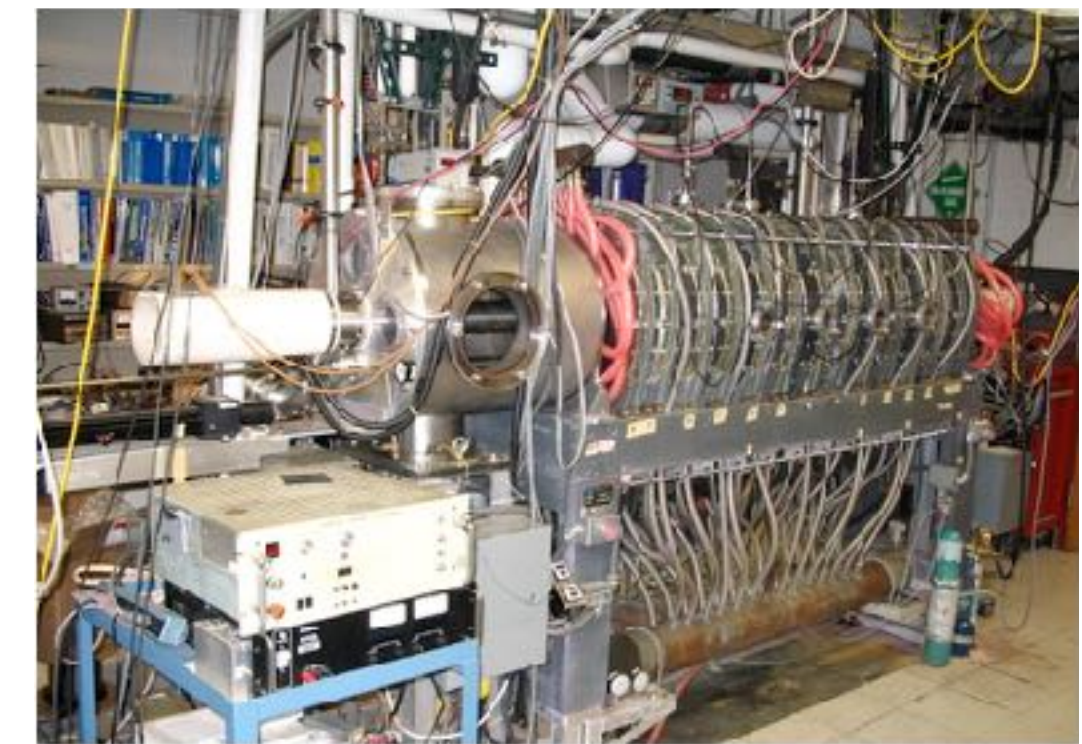
sun



and interior of stars, supernovas,
and white dwarfs

heated materials

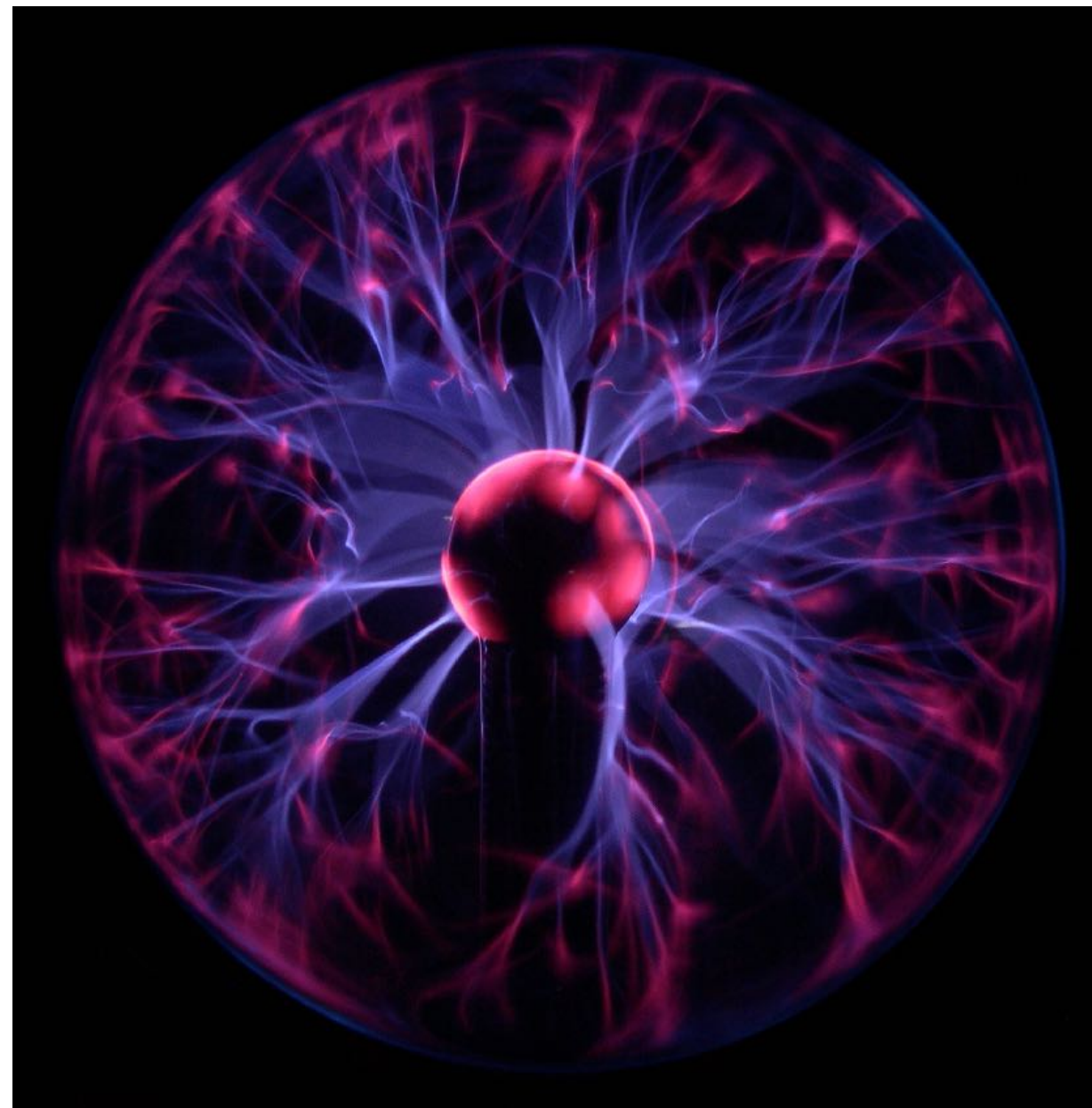
Q-machine



Examples - discharge plasmas

gas in any discharge where electrons gain sufficient energy from applied electric field thereby ionising other atoms and molecules

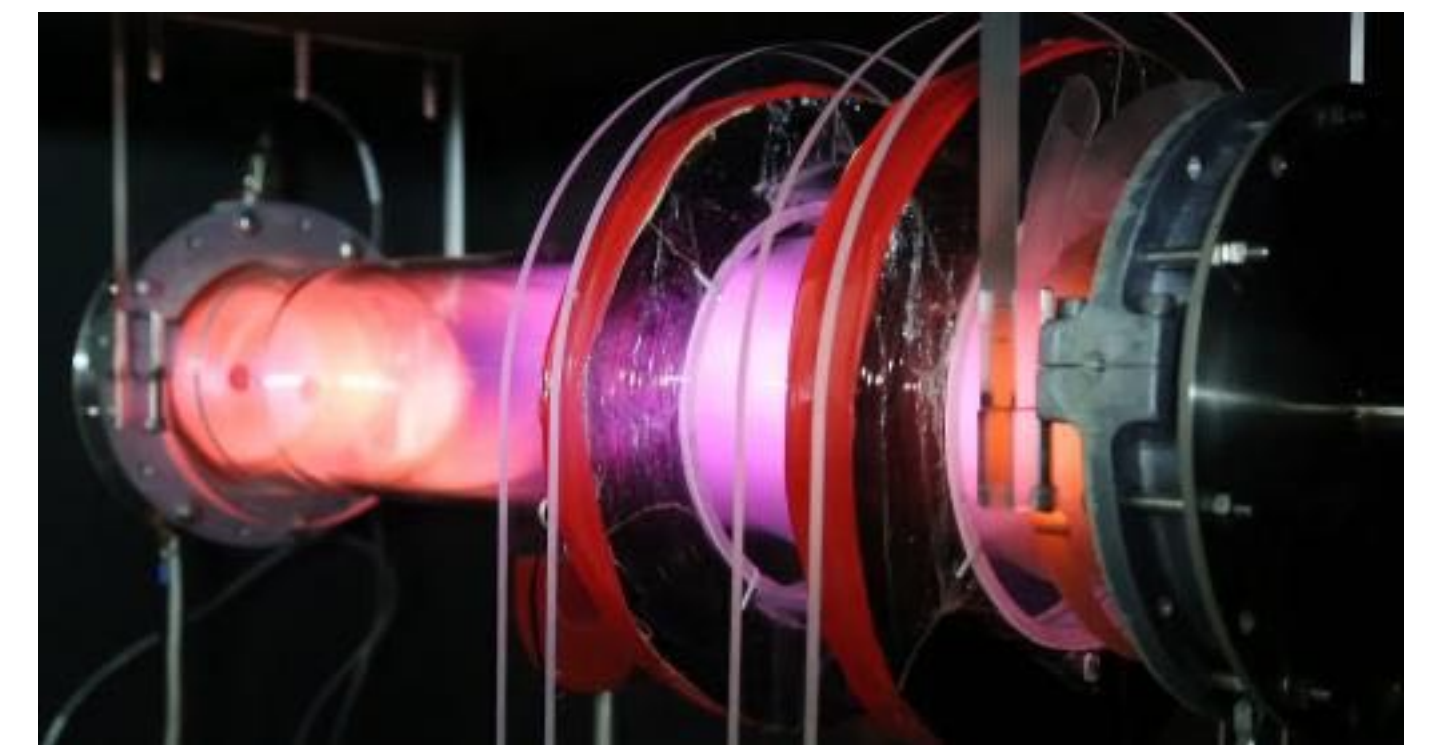
illumination



lightnings



discharge plasmas in the lab



Examples - interstellar gas

interstellar gas ionised by ultraviolet and x-rays from stars

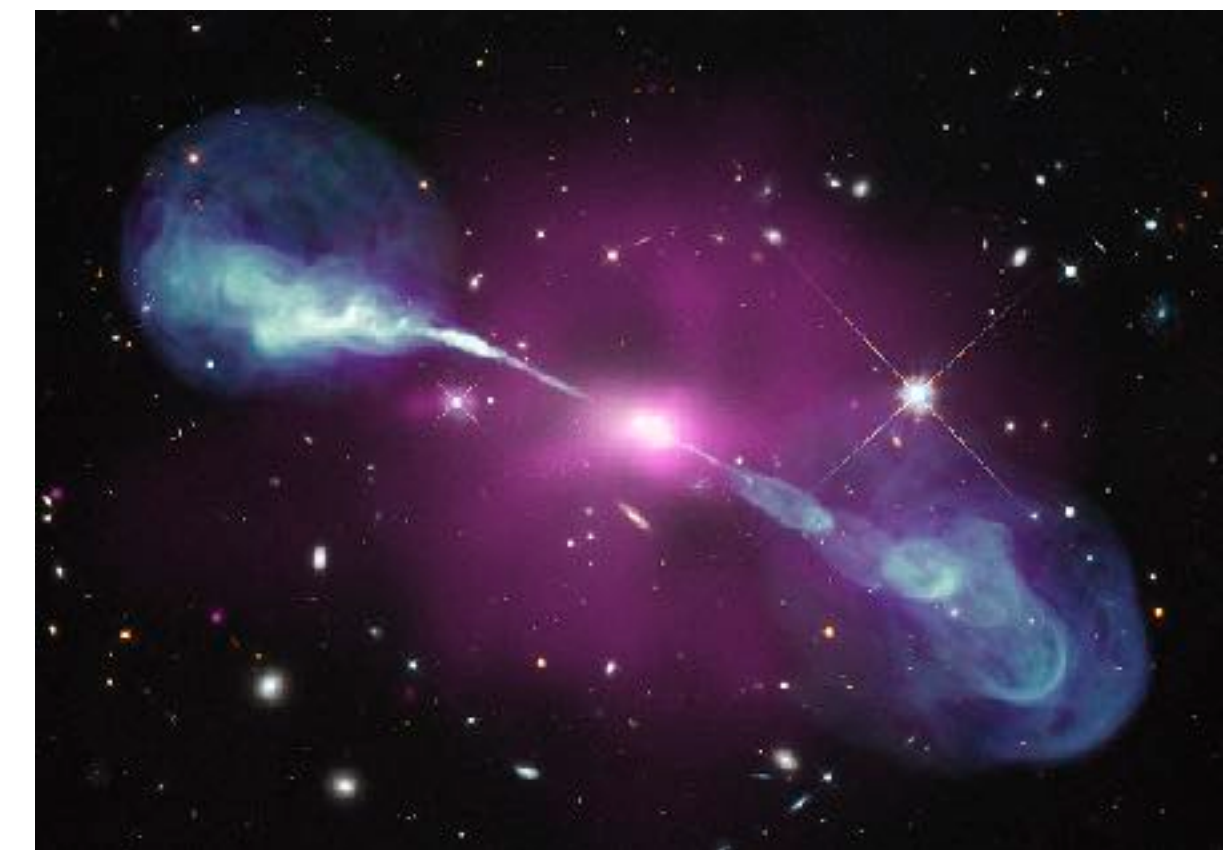
crab nebula



Cygnus A galaxy



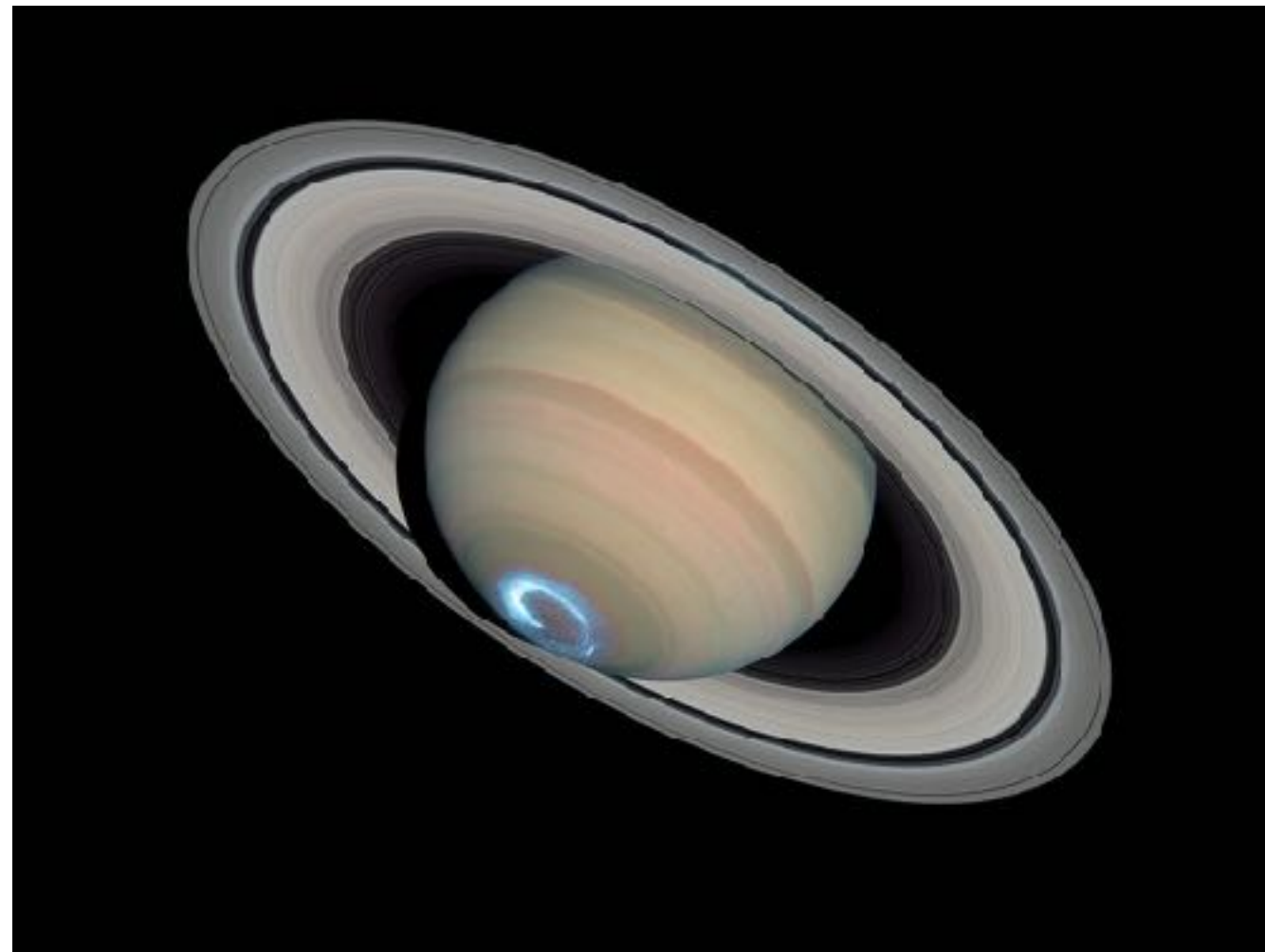
Hercules A galaxy



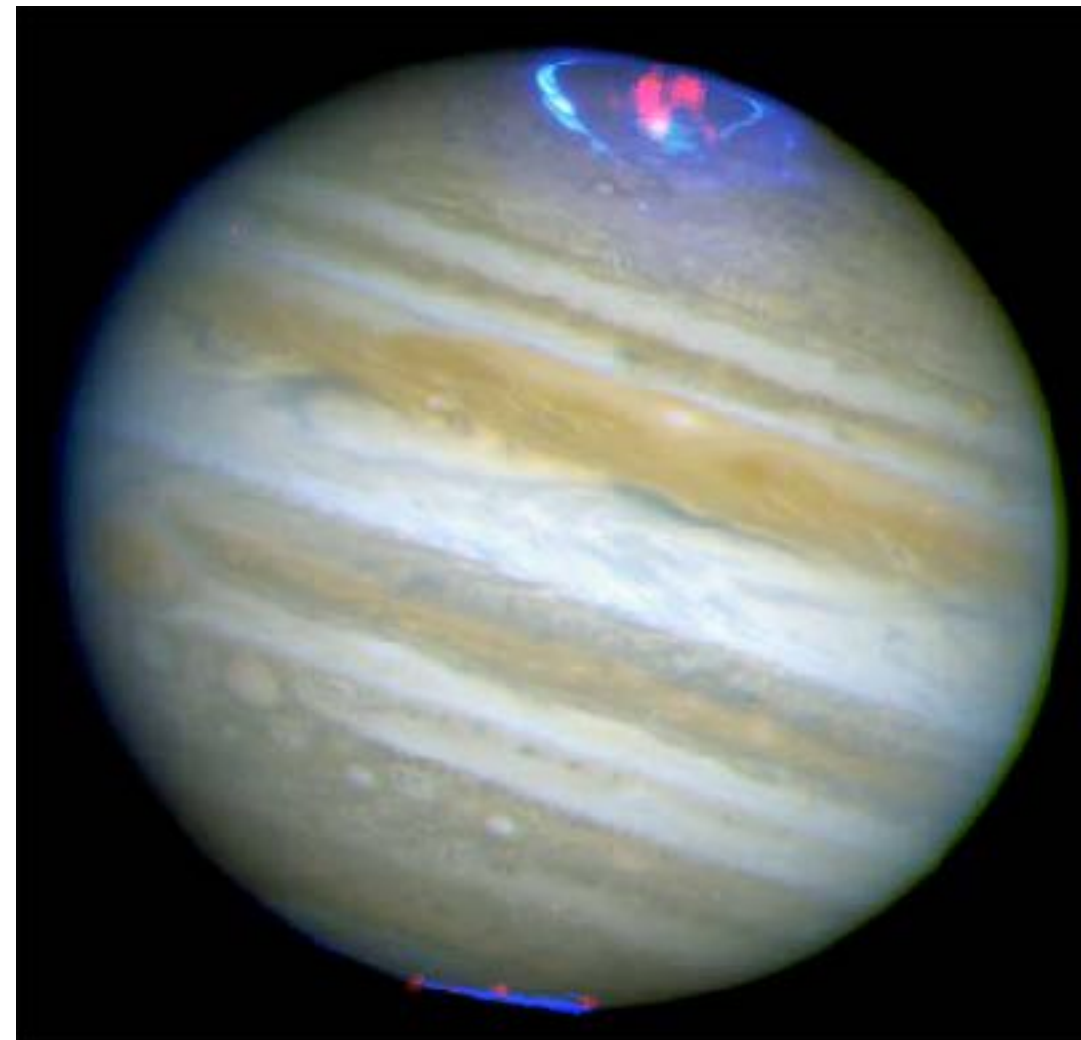
Examples - planetary plasmas

ionisation due to ultraviolet radiation from the sun

saturn's auroras



jupiter's auroras



earth's ionosphere



<https://plasma.pics>

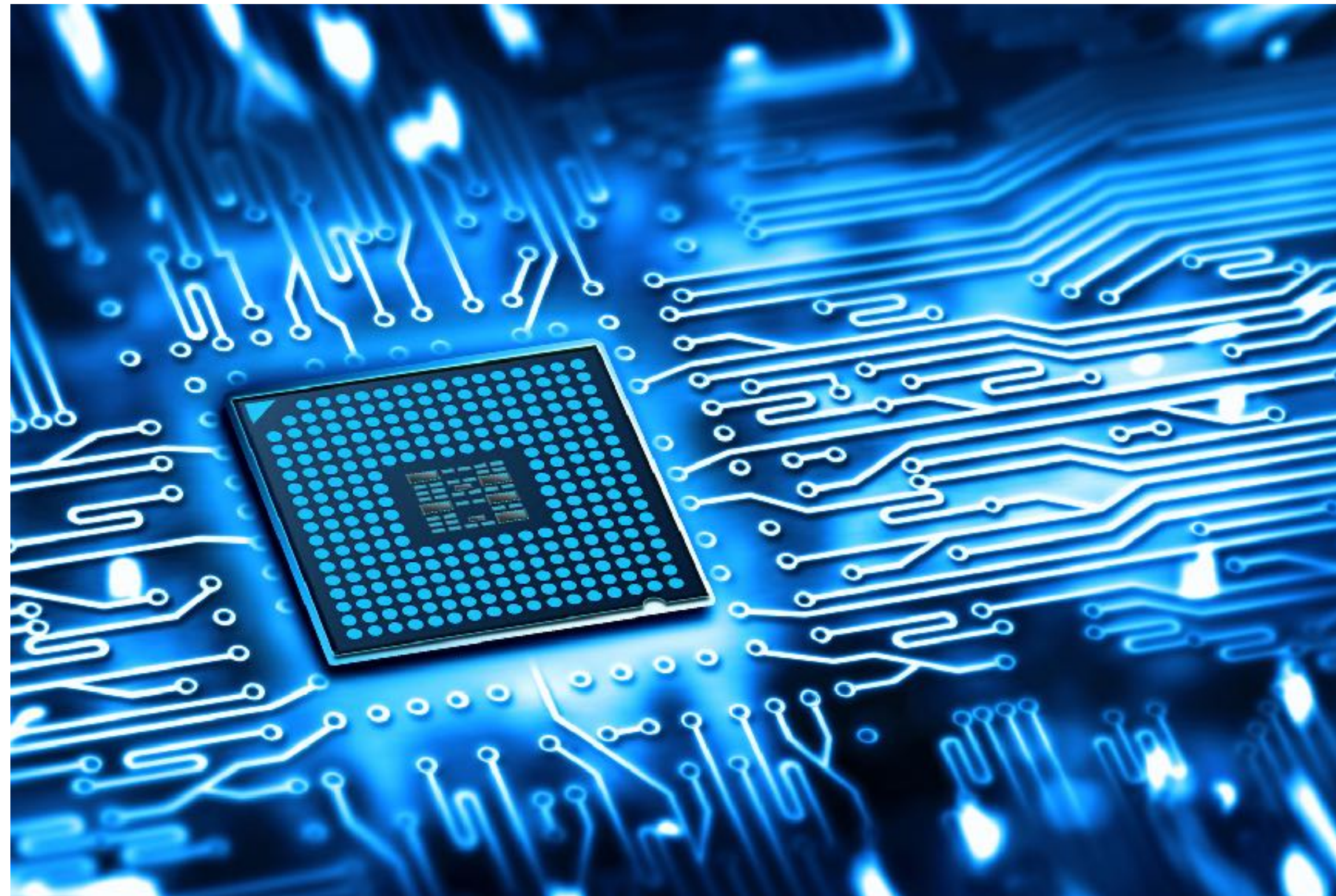
NASA

Counter examples

there are systems with mobile charges that are not plasmas



salt solutions



semiconductors

hot gases with few free
electrons, ...

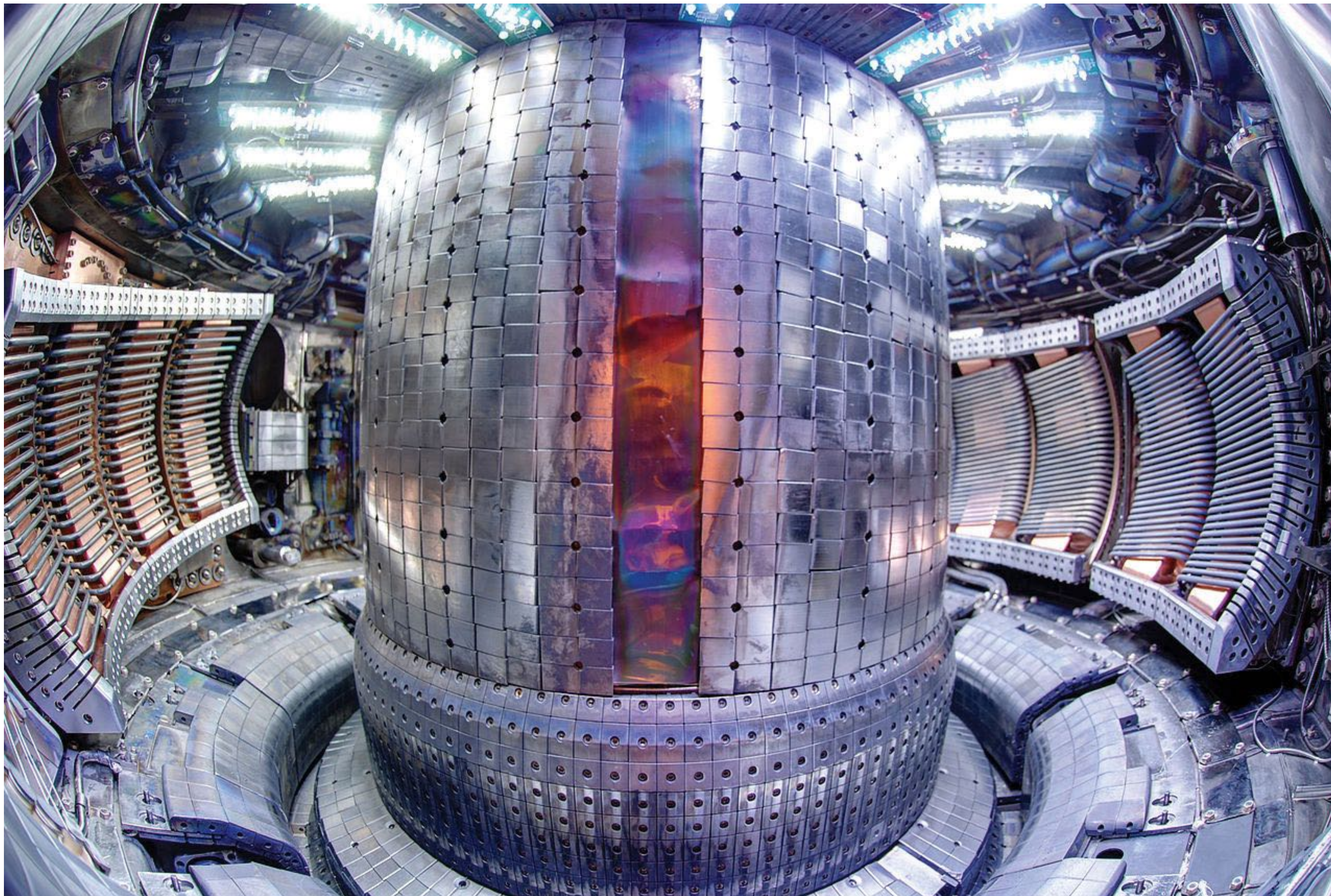
collisions with neutrals dominate and mask collective dynamics

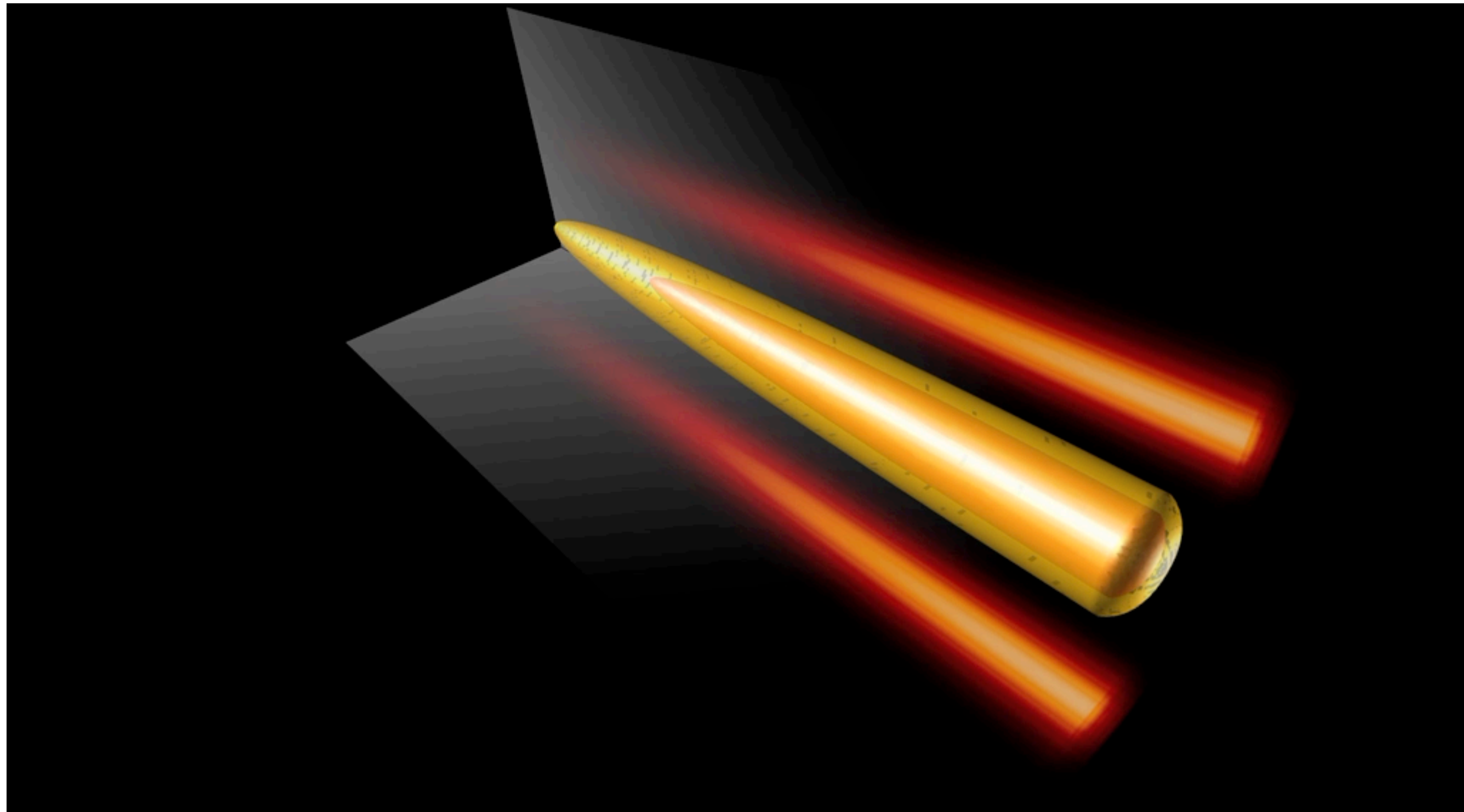
Plasmas are the fourth state of matter



- no sharp transition between gas and plasma (e.g. gas can be continuously heated thereby increasing ionisation level progressively)
- but the physics is dramatically different in the plasma state because particles interact much strongly (with electric and magnetic fields) than non-conducting gases and fluids
- wealth of phenomena not exhibited by non-plasmas and richer physics than air, or water

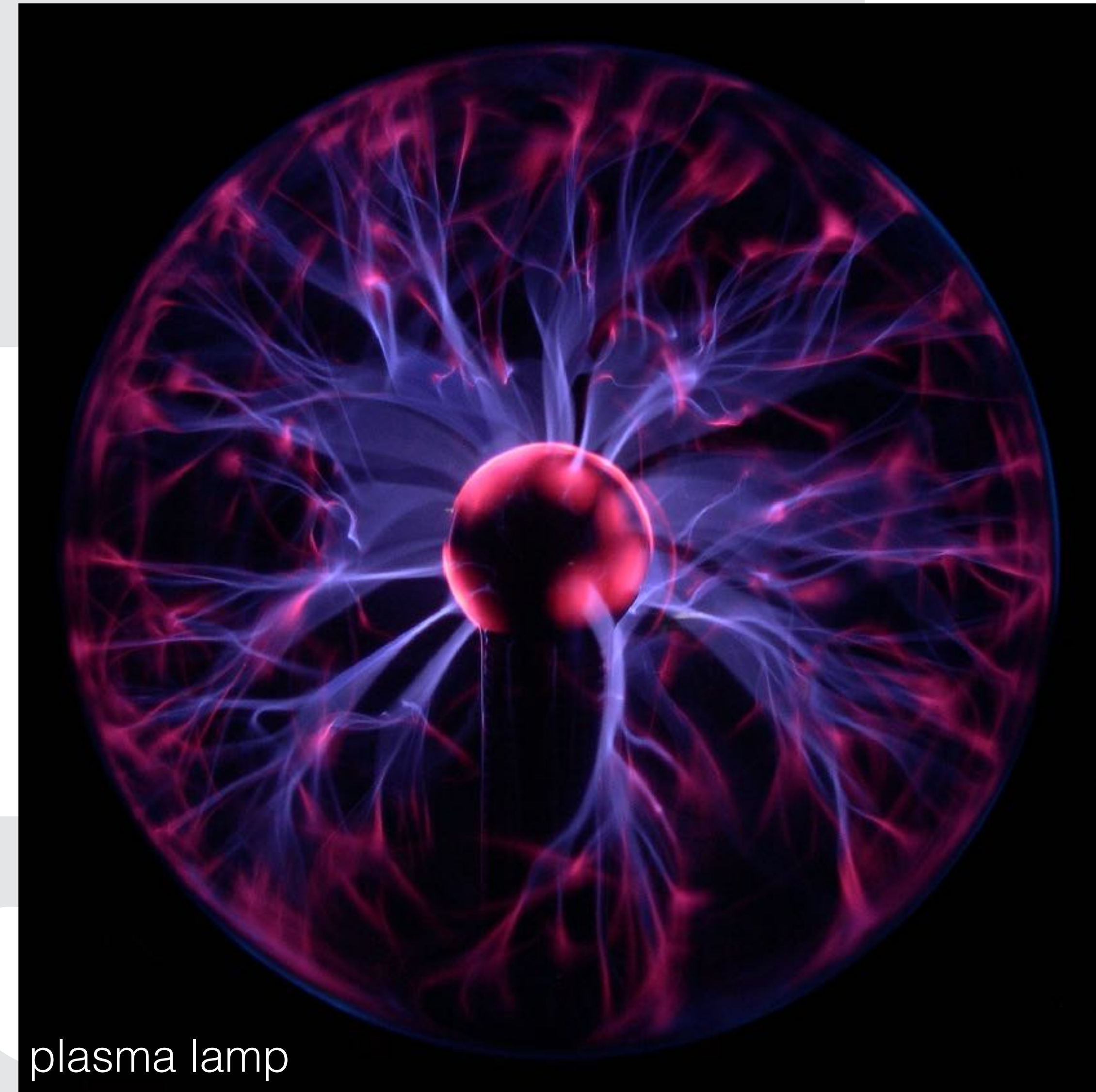
Applications - fusion energy





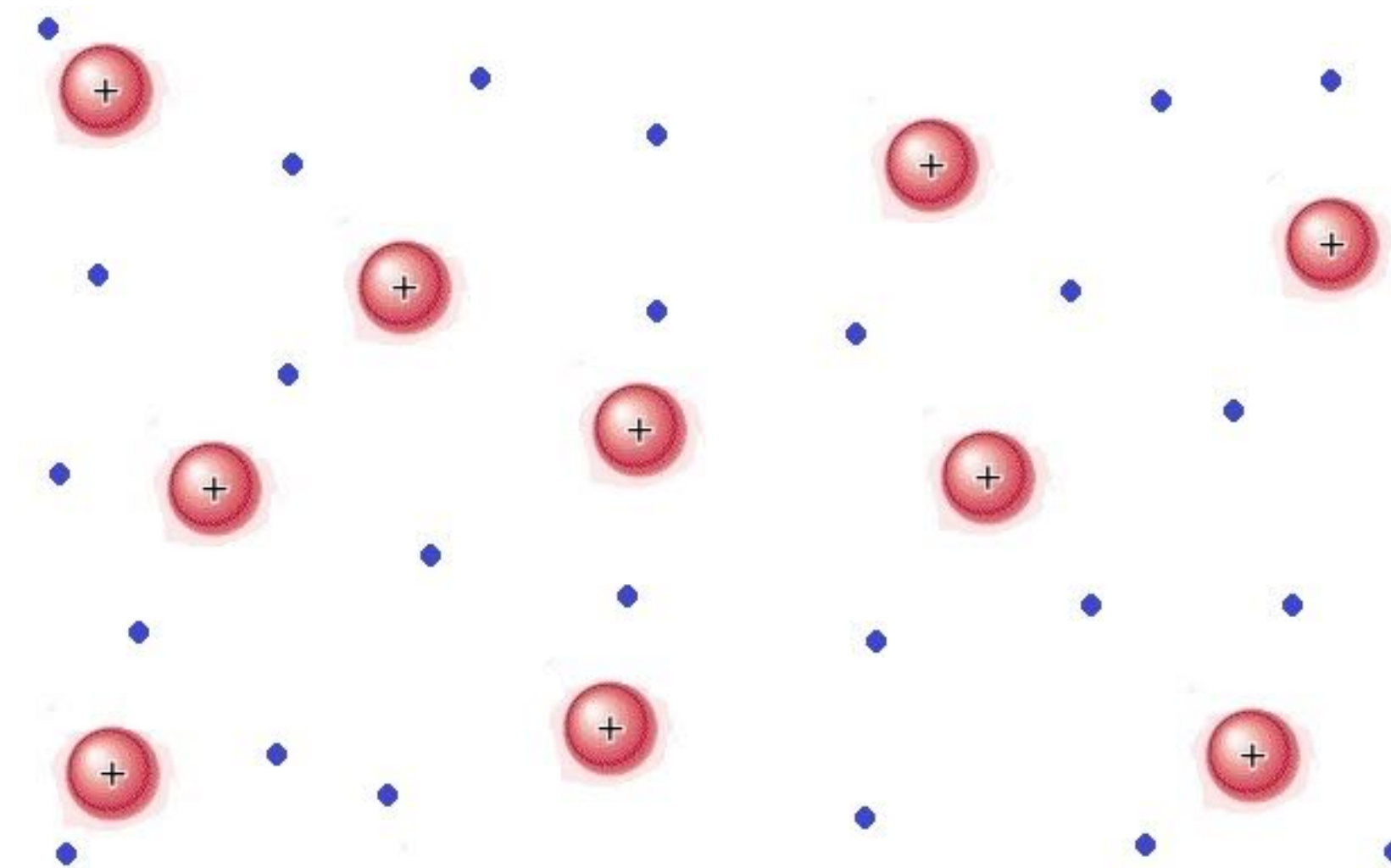
- plasma propulsion
- micro and nanoelectronics
- de-pollution
- biology and medicine
- processing of materials
- QED processes
- ...

- general info
 - evaluation
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- **basic concepts**
 - Debye shielding
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Debye shielding - kinetic vs potential energy balance

- Typical temperature of a plasma is of the order (0.01-1) of the ionisation potential (eV)
- Example: plasma in a Q-machine
 - $n_0 \sim 10^{12} \text{ cm}^{-3}$
 - $T \sim 2500^\circ \text{ K} = 0.22 \text{ eV}$
 - Ionisation degree $\sim 100 \%$
- mean distance between electron and ion
 $r \sim 10^{-4} \text{ cm}$

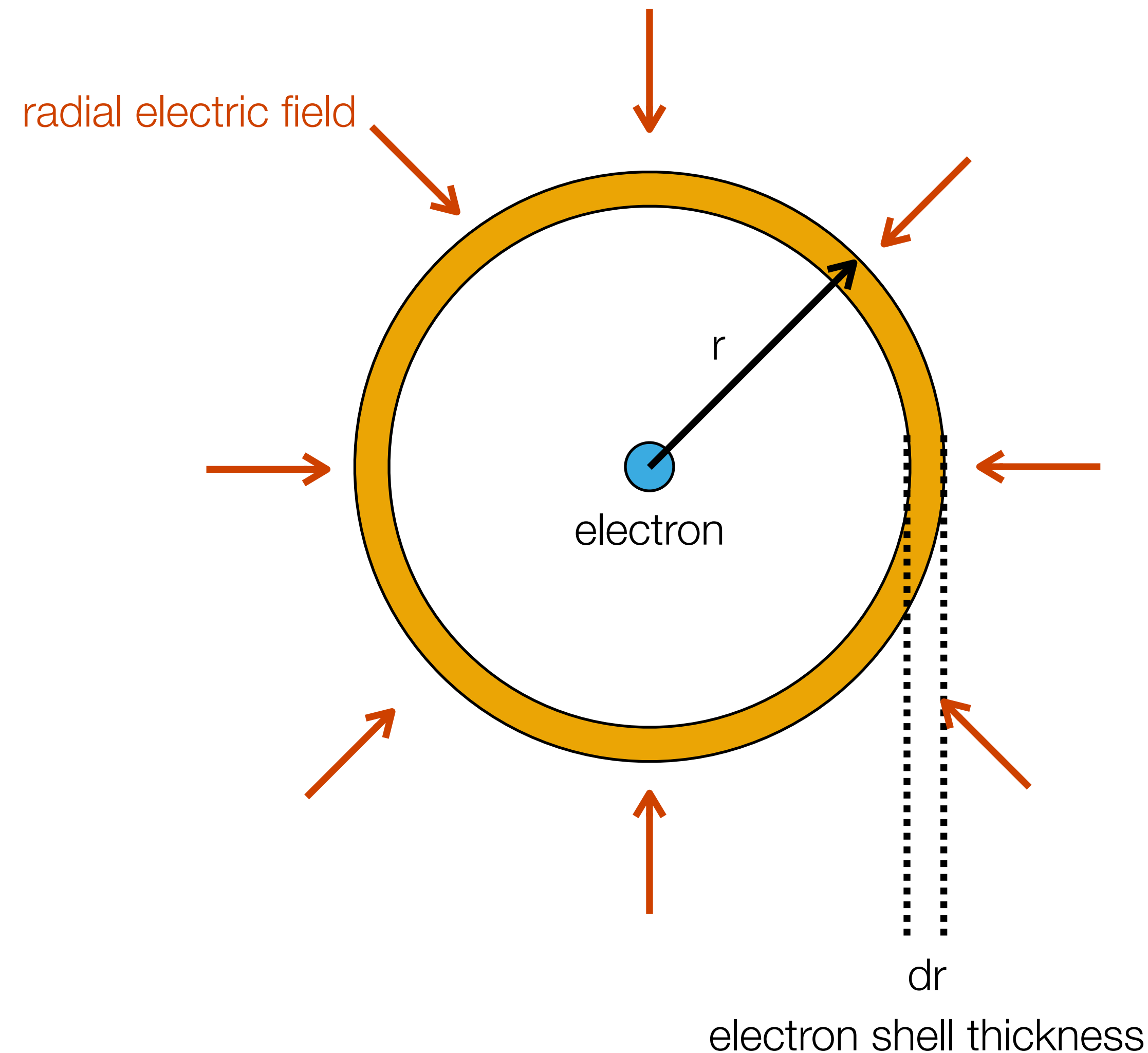


electron (and ion) potential energy is much smaller than electron (and ion) kinetic energy

$$e\phi = \frac{e^2}{4\pi\epsilon_0 r} = \frac{(1.6 \times 10^{-19} \text{ C})^2}{4\pi \times 8.9 \times 10^{-12} \text{ Farad/m} \times 10^{-6} \text{ m}} = 2.3 \times 10^{-22} \text{ J} = 1.4 \times 10^{-3} \text{ eV}$$

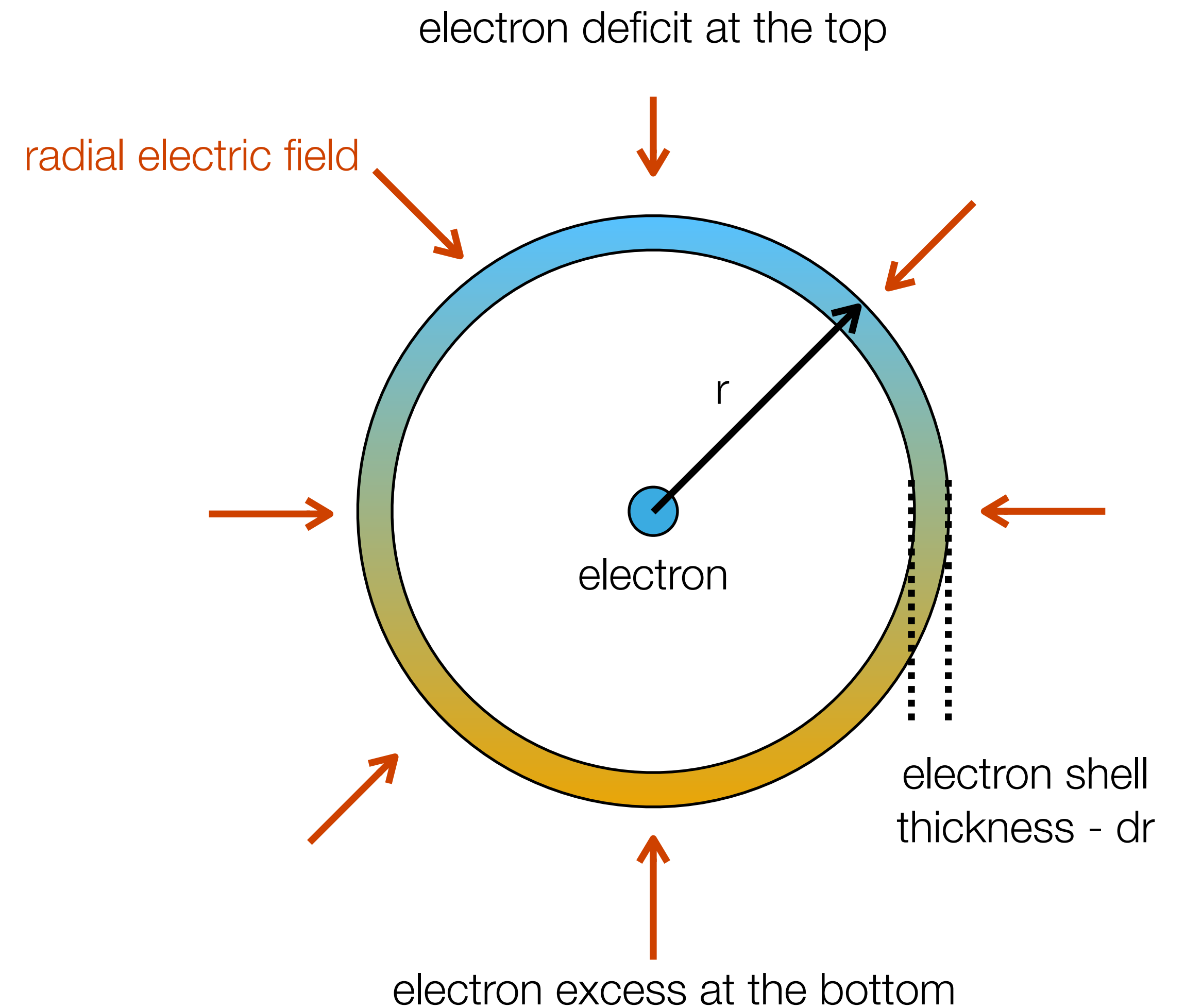
Debye shielding - long range interactions

- interaction between pairs of particles is very small
- but: sum of the fields of many ions and electrons can be important because of the long range nature of Coulomb interactions
- Example: charged electron sphere
 - electron number in shell $dN = 4\pi n_e r^2 dr$
 - E-field of each electron in shell $\propto r^{-2}$
 - electric field of shell is independent of r
- Uniformly charged sphere the total E field at the electron location is zero



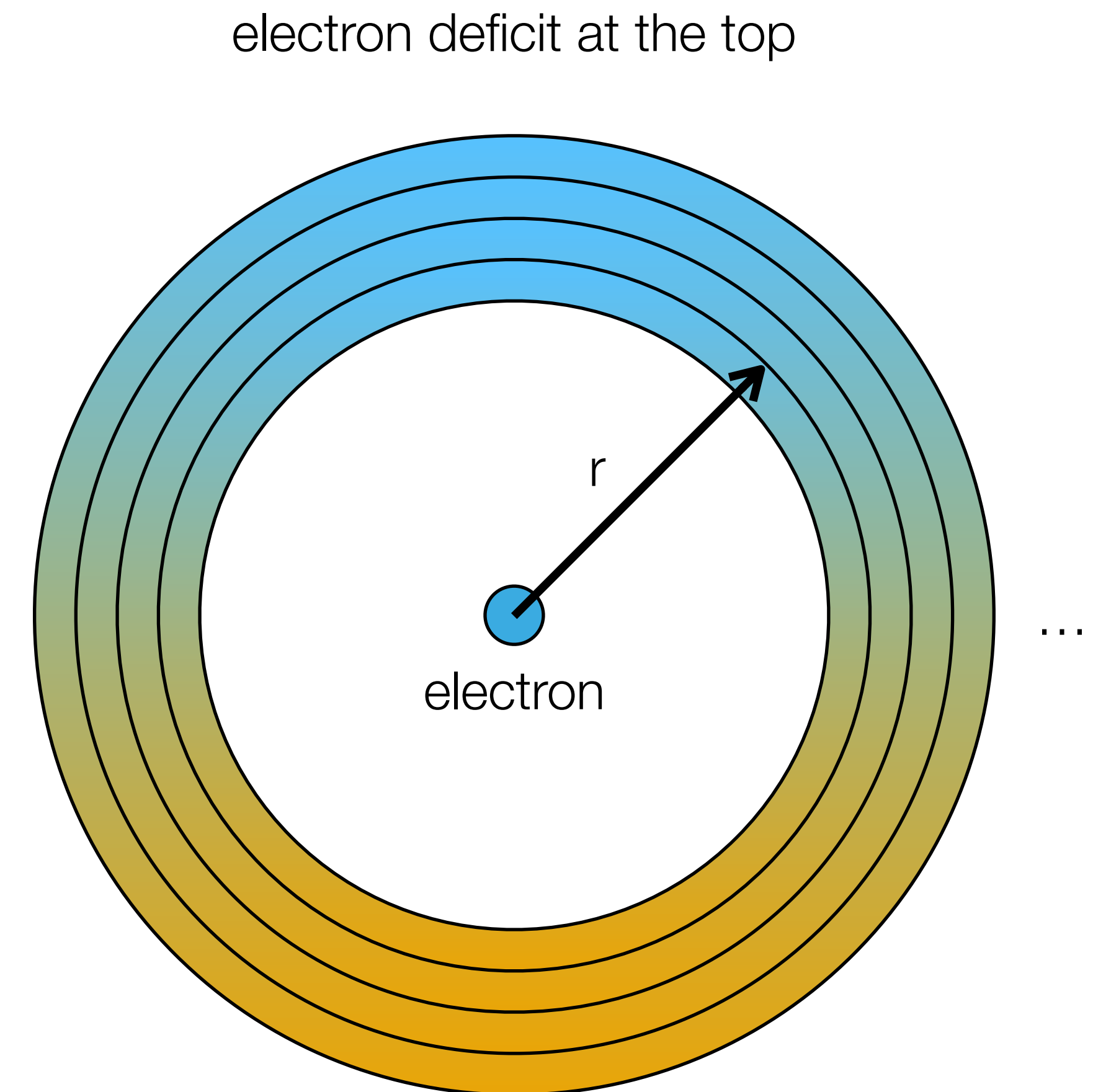
Debye shielding - long range interactions

- displace e.g. 1% of the electron charge from up to down



Debye shielding - long range interactions

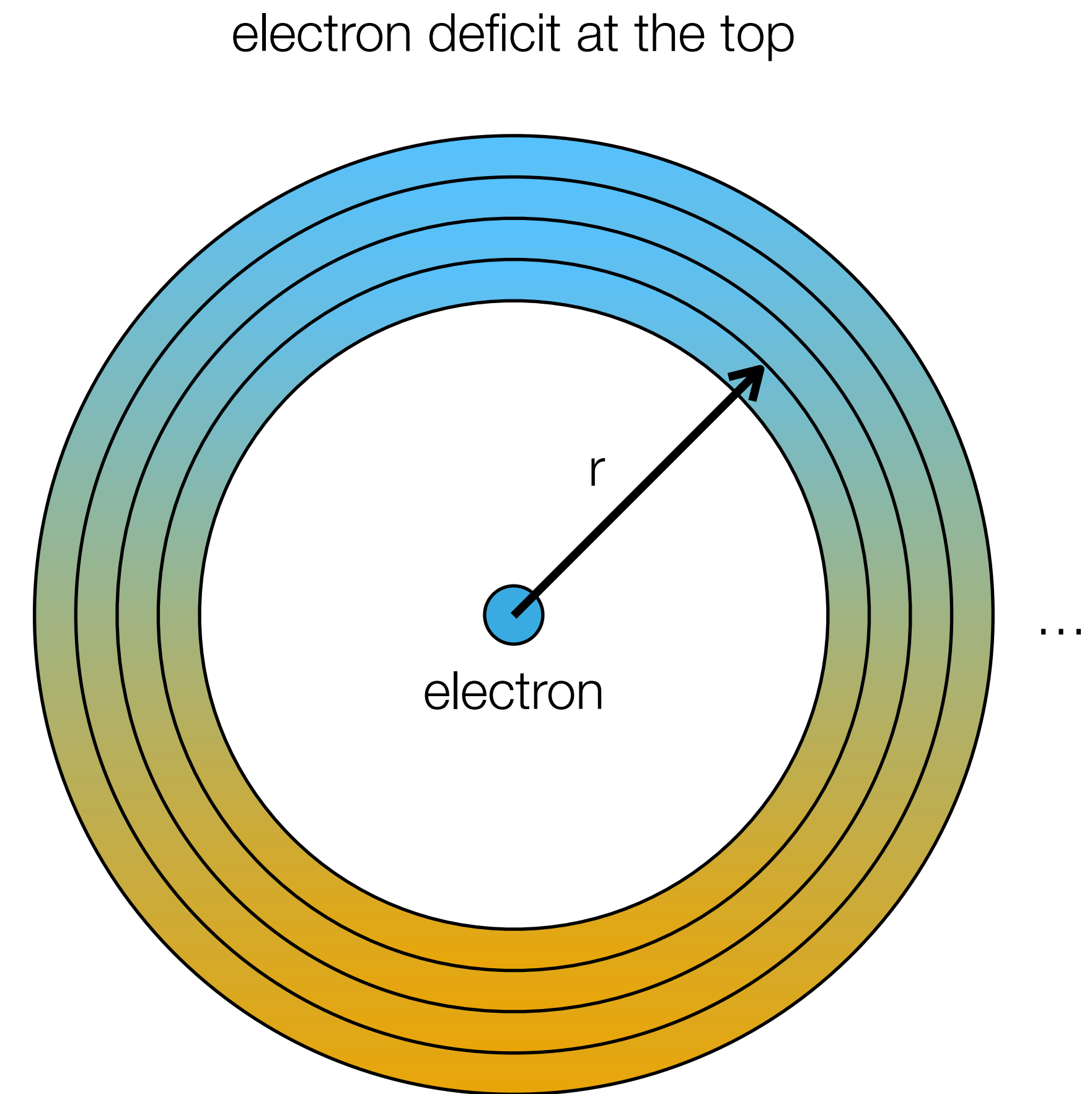
- displace e.g. 1% of the electron charge from up to down
- repeat this for many spherical shells over a large distance
- very large field is produced at the electron location because every shell will contribute to the electric field as any other shell



radial electric field of every shell at the electron location is the same

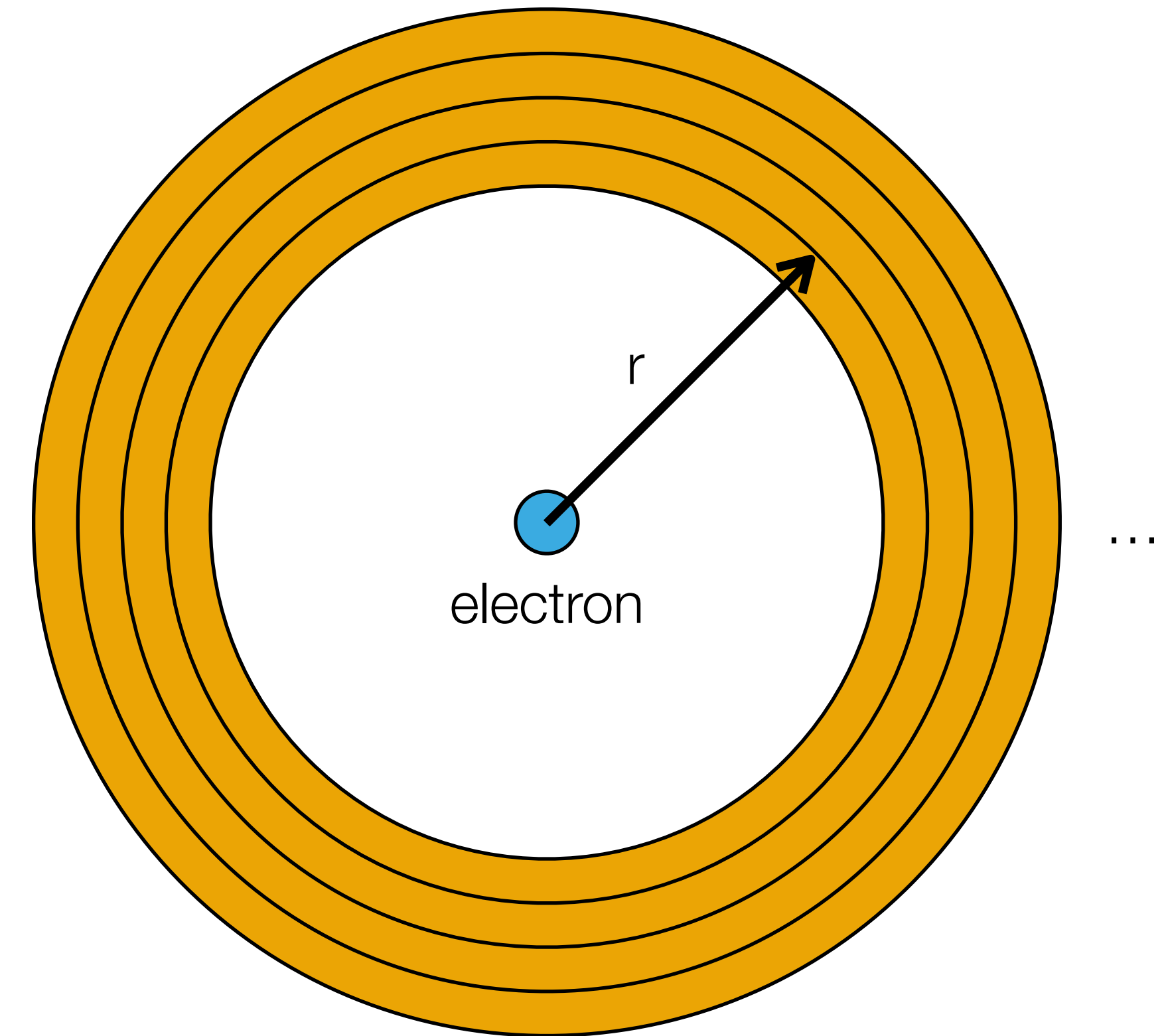
Debye shielding - long range interactions

- electrons and ions quickly move towards the regions where there are excess electrons and excess ions



radial electric field of every shell at the electron location is the same

- electrons and ions quickly move towards the regions where there are excess electrons and excess ions
- electrons are light and do not stop when charge balance is established

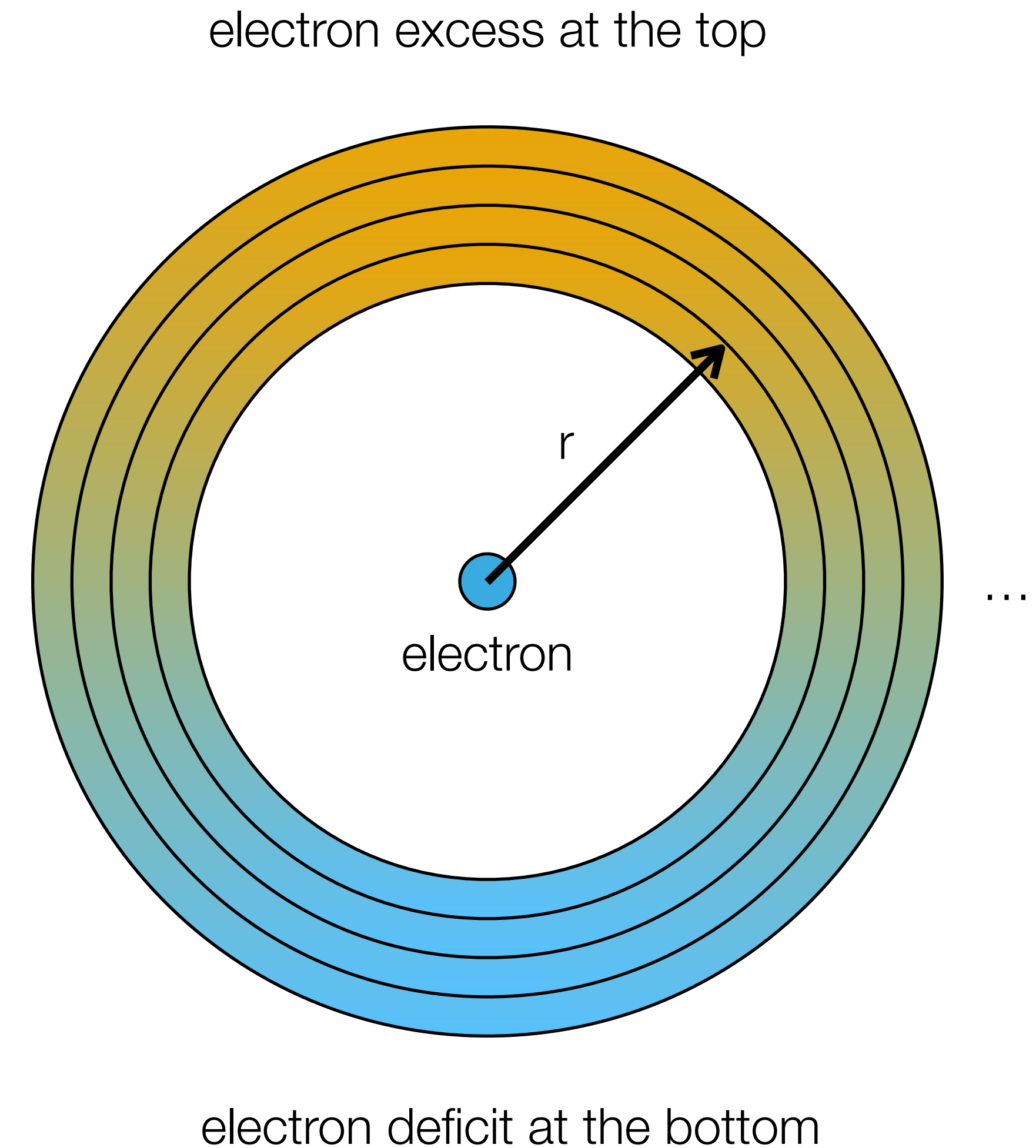


no excess/deficit

radial electric field of every shell at the
electron location is the same

Debye shielding - long range interactions

- electrons and ions quickly move towards the regions where there are excess electrons and excess ions
- electrons are light and do not stop when charge balance is established
- the charge imbalance reverses until the electric field stops the motion



radial electric field of every shell at the electron location is the same

plasma oscillation at the plasma electron frequency (more details ahead)

$$\omega_p^2 = \frac{4\pi n_e e^2}{m_e} \text{ [CGS]} = \frac{n_e e^2}{\epsilon_0 m_e} \text{ [SI]} \sim 6 \times 10^4 (n_e [\text{cm}^{-3}])^{1/2}$$

ω_p is in rad s^{-1}

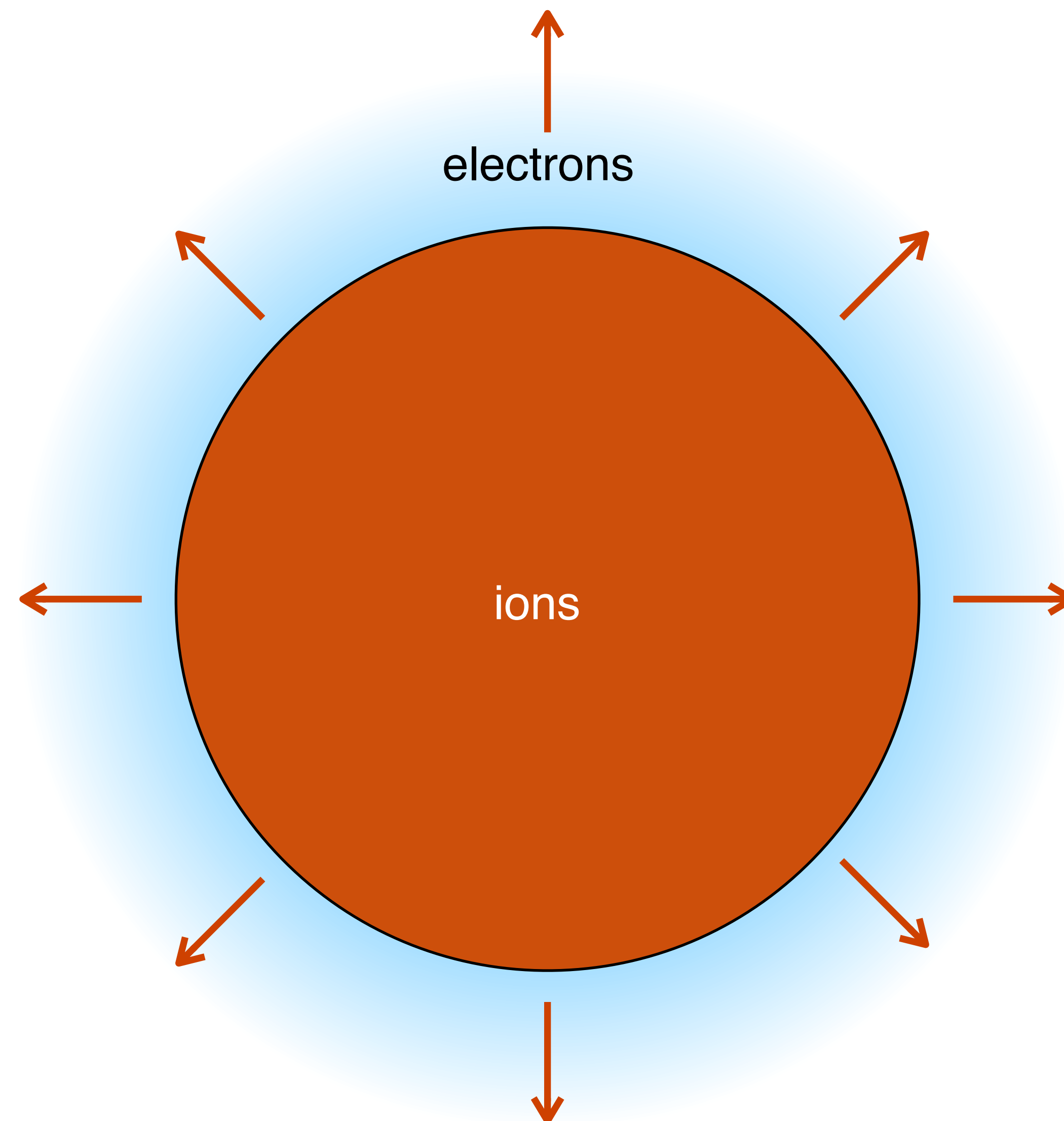
Debye shielding - how strong are space charge effects

how much energy would it take to remove all the electrons from a spherical region of radius r ?

$$W = \int \frac{\epsilon_0 E^2}{2} dV$$

...

$$W = \frac{2\pi}{45} \frac{n_0^2 e^2 r^5}{\epsilon_0}$$



Debye shielding - importance of space charge effects

What is the radius of the sphere where the electron thermal energy is enough to remove themselves from the sphere?

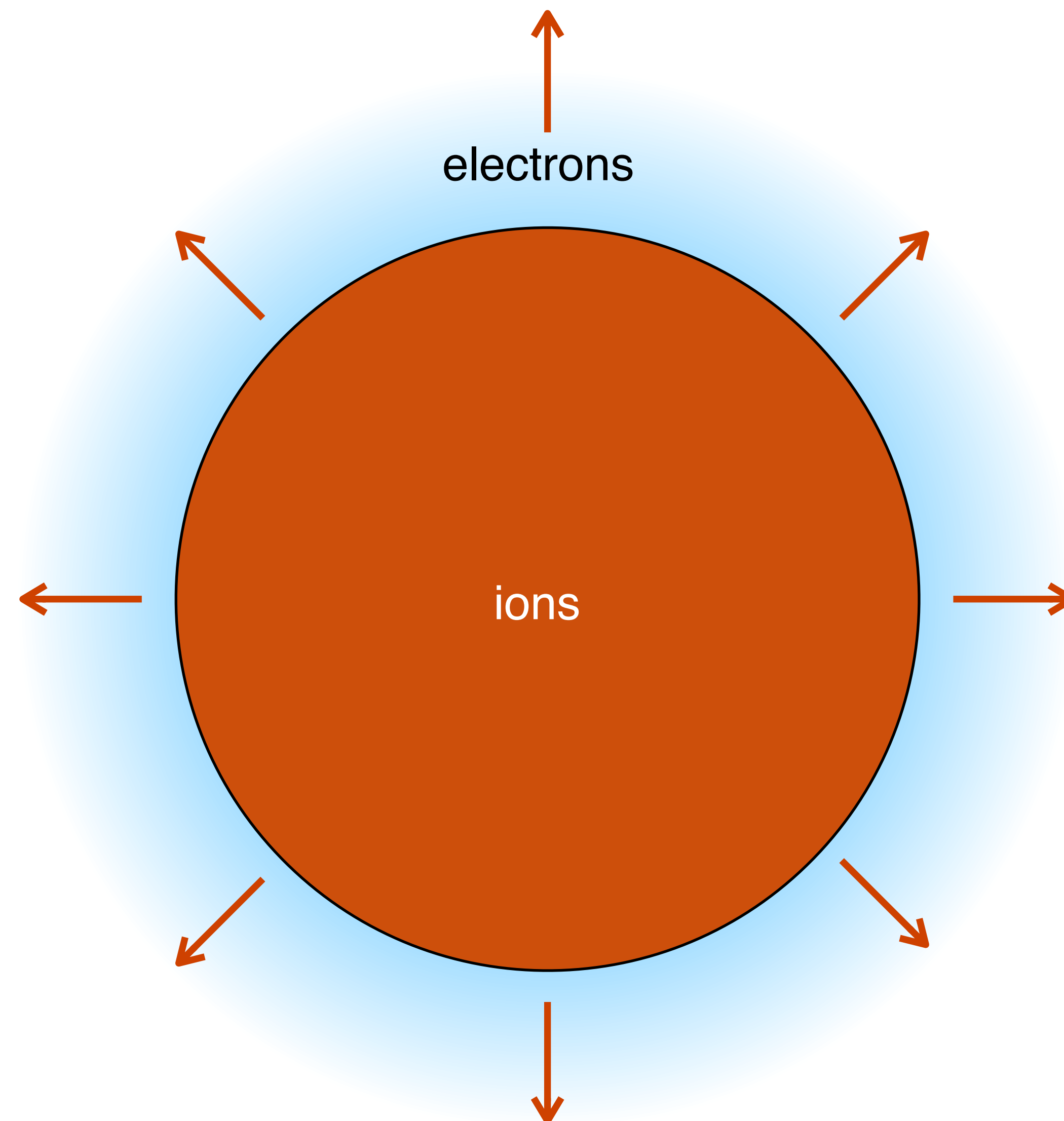
$$r_s = 45 \frac{\epsilon_0 k T}{n_e e^2}$$

Debye length

$$\lambda_D^2 = \frac{\epsilon_0 k T}{n_e e^2} = \frac{v_T^2}{\omega_p^2}$$

thermal velocity

$$v_T^2 = \frac{k T}{m}$$



- charge imbalances are much smaller for distances much higher than r_s because there is not enough energy to separate electrons from ions.
- large charge imbalances can occur for distances smaller than r_s because the electron thermal energy is sufficient to remove all electrons themselves from the sphere.
- the Debye length is thus a measure of the size of a region in which appreciable deviations from charge neutrality occur in a thermal plasma.
- as electrons move they push the plasma electrons away. as they move a Debye length, the plasma has time to neutralise its electric field and recover neutrality

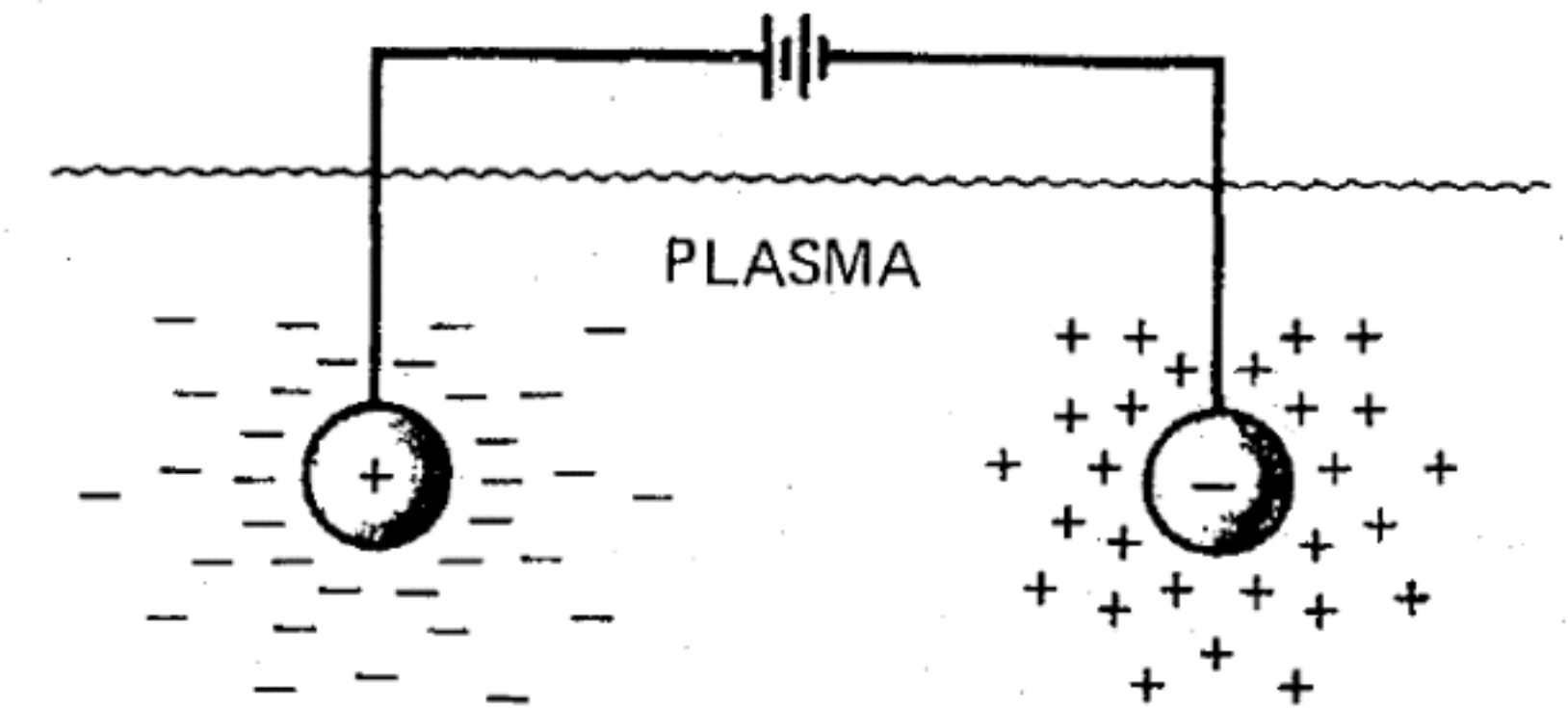
the Debye length is much larger than the inter-particle distance $d \propto n_0^{-1/3}$

$$\frac{\lambda_D^2}{d^2} = \frac{\epsilon_0 k T d^3}{d^2 e^2} = \frac{\epsilon_0 k T}{e^2 / d} = \frac{k T}{e \phi(d)} \gg 1$$

- **the number of particles per Debye length in a plasma is very large.**
- plasma parameter, Λ , is the number of particles in a Debye sphere must be much larger than unity in a plasma

$$\Lambda = n_e \lambda_D^3 \gg 1$$

- test charge: positive ion, infinite mass
- attracts electrons, repels positive ions
- electron density increases in the vicinity of the charge
- a negative electron cloud (shield) tends to cancel test charge



Poisson equation

$$\nabla^2 \phi = -\frac{e}{\epsilon_0} [n_i - n_e + q_t \delta(\mathbf{r})]$$

$$\nabla^2 = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr} \right)$$

- e is the elementary charge
- q_t is a point test charge
- n_i and n_e are the ion and electron densities

distribution function

$$n_{i,e} = n_0 \exp \left(\frac{e\phi}{kT_{i,e}} \right)$$

- statistical mechanics equilibrium
- n_i and n_e are the ion and electron densities

electrostatic potential

$$\phi = \frac{q_t}{r} \exp \left(-\frac{r}{\lambda_D} \right)$$

$$\lambda_D^{-2} = \lambda_{D,i}^{-2} + \lambda_{D,e}^{-2}$$

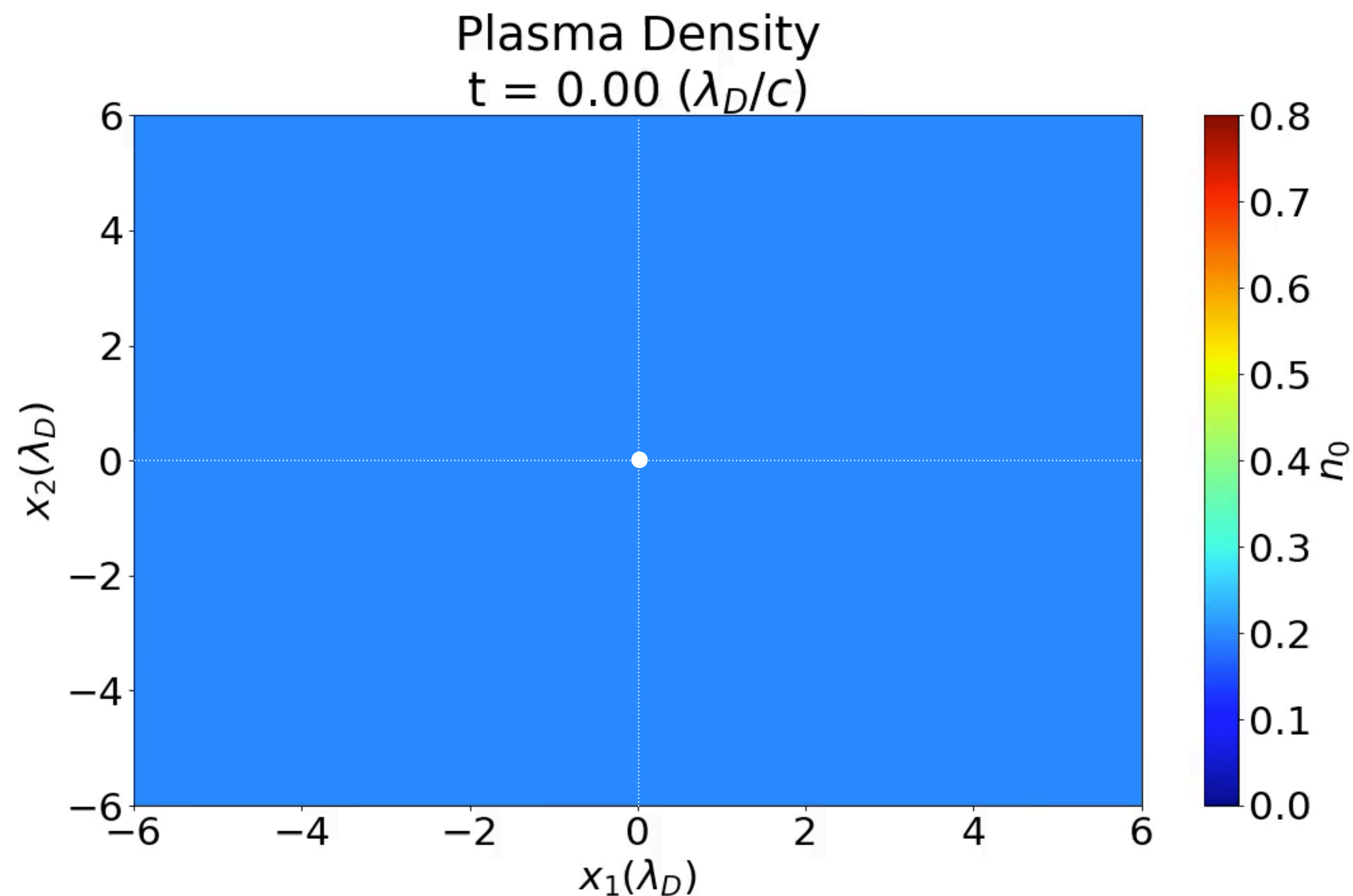
$$\lambda_{D,i,e} = \left(\frac{\epsilon_0 kT_{i,e}}{n_0 e^2} \right)$$

typical values for the Debye length

Plasma	Density $n_e(\text{m}^{-3})$	Electron temperature $T(\text{K})$	Magnetic field $B(\text{T})$	Debye length $\lambda_D(\text{m})$
Solar core	10^{32}	10^7	—	10^{-11}
Tokamak	10^{20}	10^8	10	10^{-4}
Gas discharge	10^{16}	10^4	—	10^{-4}
Ionosphere	10^{12}	10^3	10^{-5}	10^{-3}
Magnetosphere	10^7	10^7	10^{-8}	10^2
Solar wind	10^6	10^5	10^{-9}	10
Interstellar medium	10^5	10^4	10^{-10}	10
Intergalactic medium	1	10^6	—	10^5

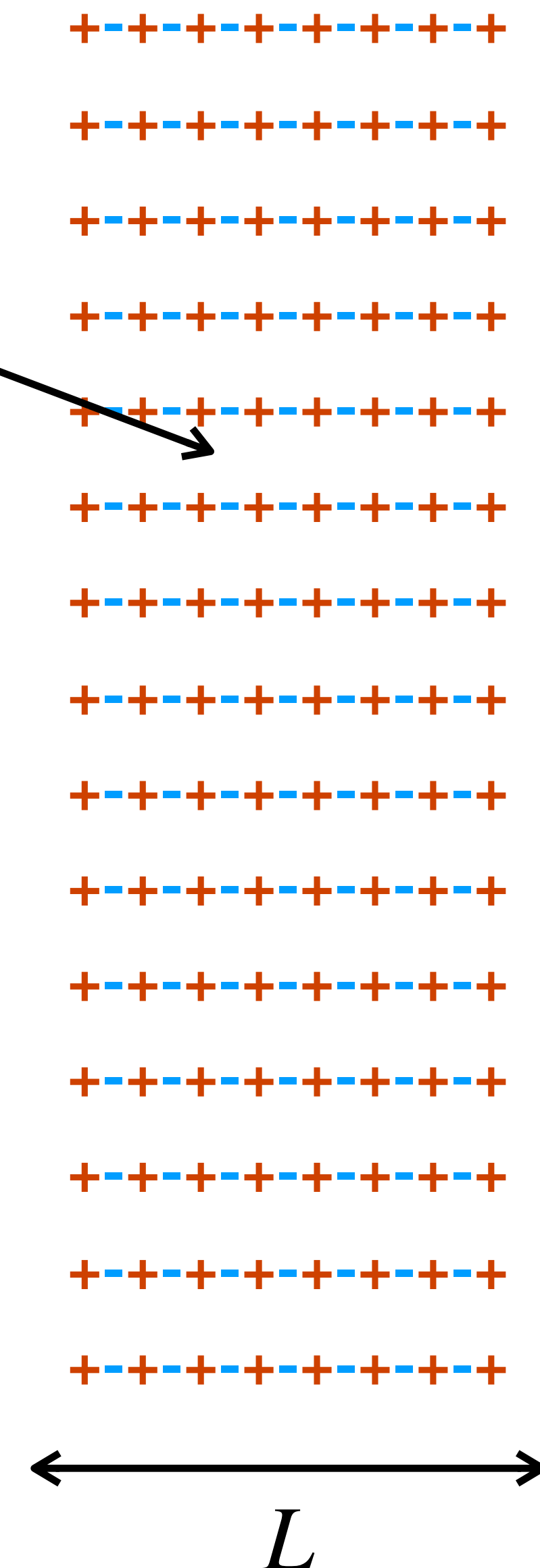
Debye shielding - computational example

- first principle simulation (captures the single particle motion of plasma)
- initial charge is a circle with radius $0.2 \lambda_D$
- ions are immobile
- plasma electrons accumulate near the charge in order to shield its charge

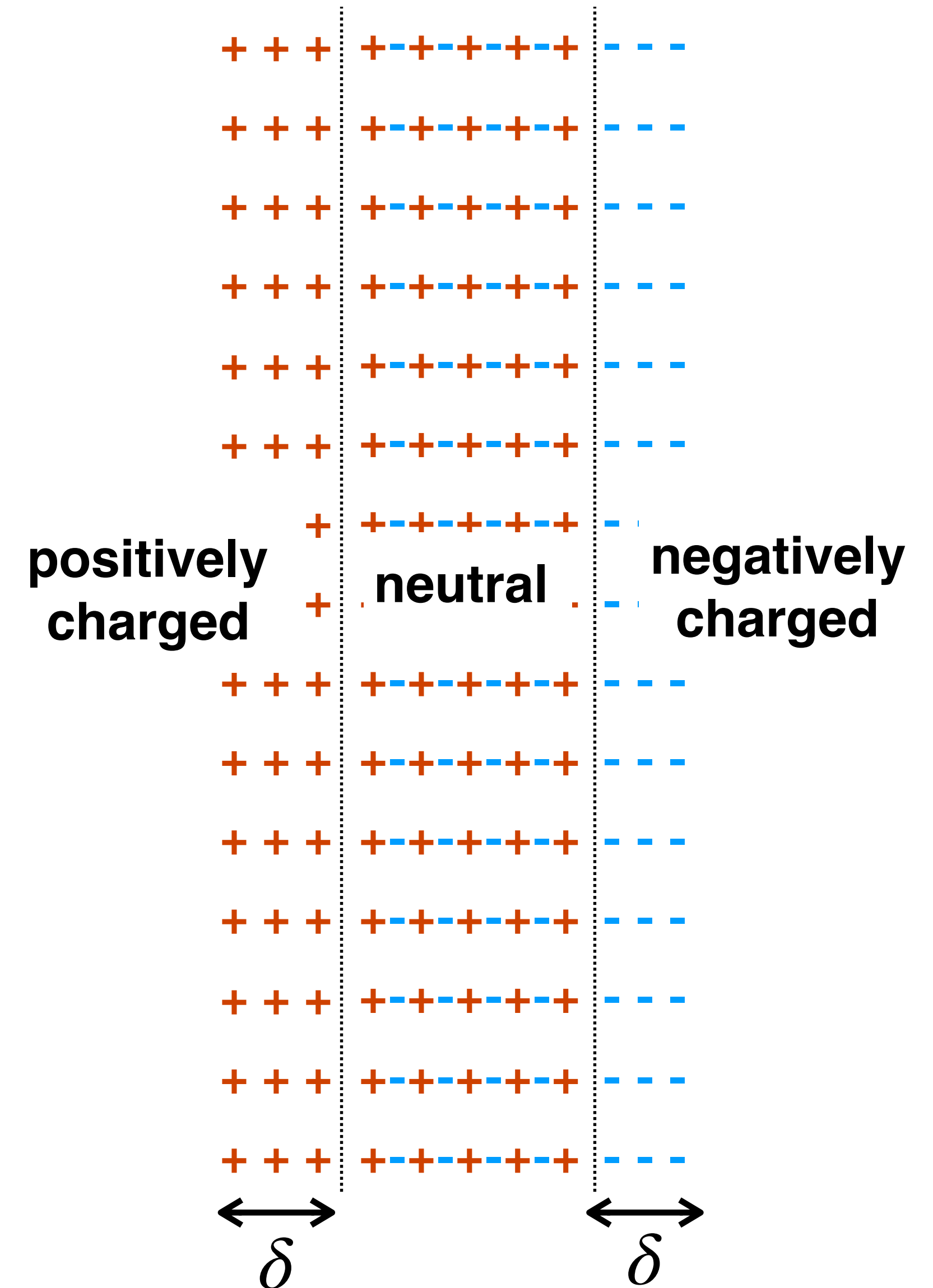


- plasma slab
 - thickness, L
 - electron and ion density, n_0
 - total charge density is zero initially
 - ions are immobile
 - relative distance between electrons is fixed but they can move freely within the ions

ion density = electron
density = n_0



- plasma slab
 - thickness, L
 - electron and ion density, n_0
 - total charge density is zero initially
 - ions are immobile
 - relative distance between electrons is fixed but they can move freely within the ions
- perturb slab
 - displace electrons (or ions) by δ
 - electric fields are that of a parallel plate capacitor

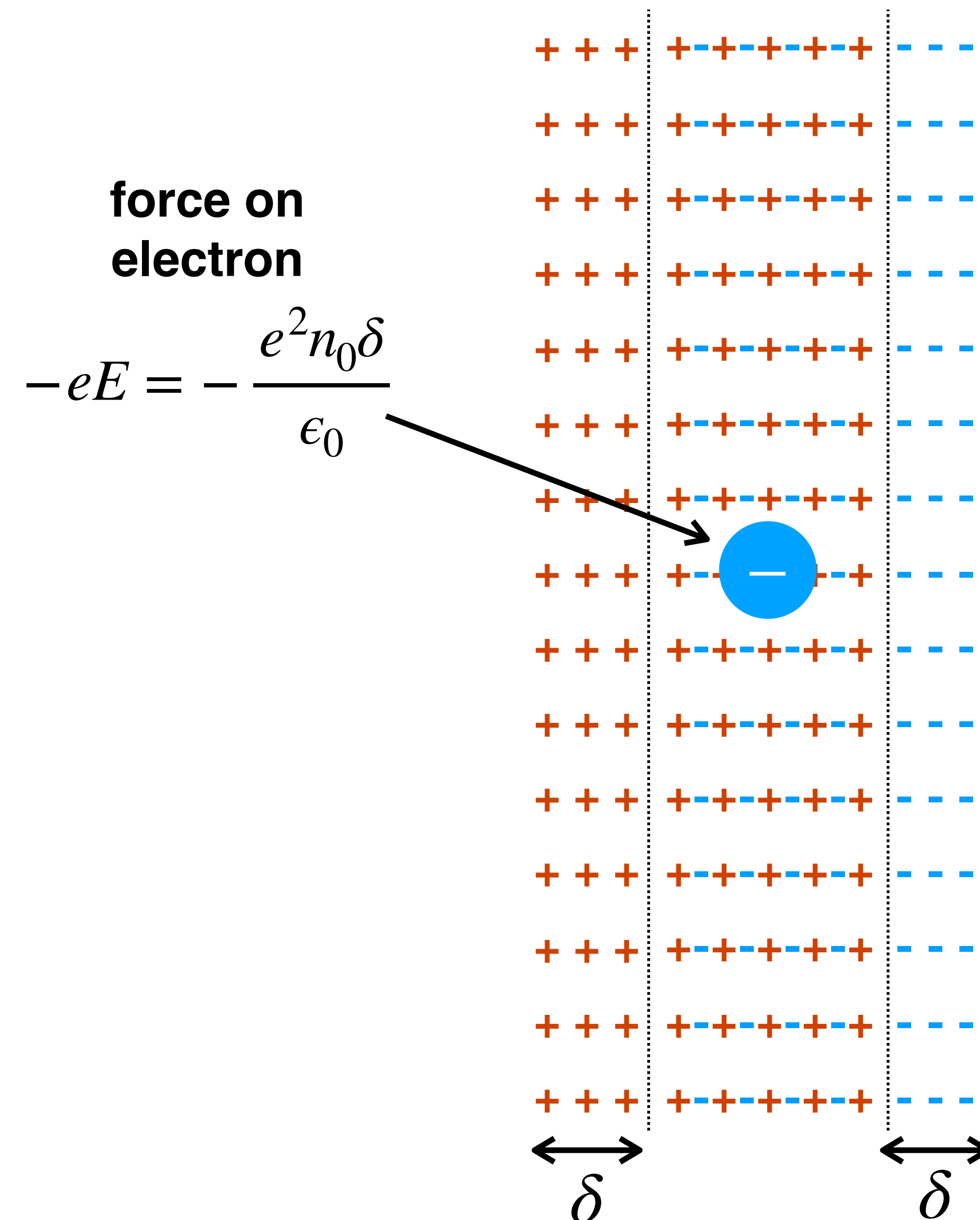


Plasma oscillations

- physical picture
 - e- slab feels a force to the left (towards ions)
 - e- velocity when e- slab overlaps with ion slab is not zero
 - e-s continue moving towards the left
 - attractive ion force pushes them back to the right
- frequency of e- slab oscillation ($\delta \ll L$)?

$$\ddot{\delta} = -\frac{e^2 n_0}{\epsilon_0 m_e} \delta = -\omega_p^2 \delta$$

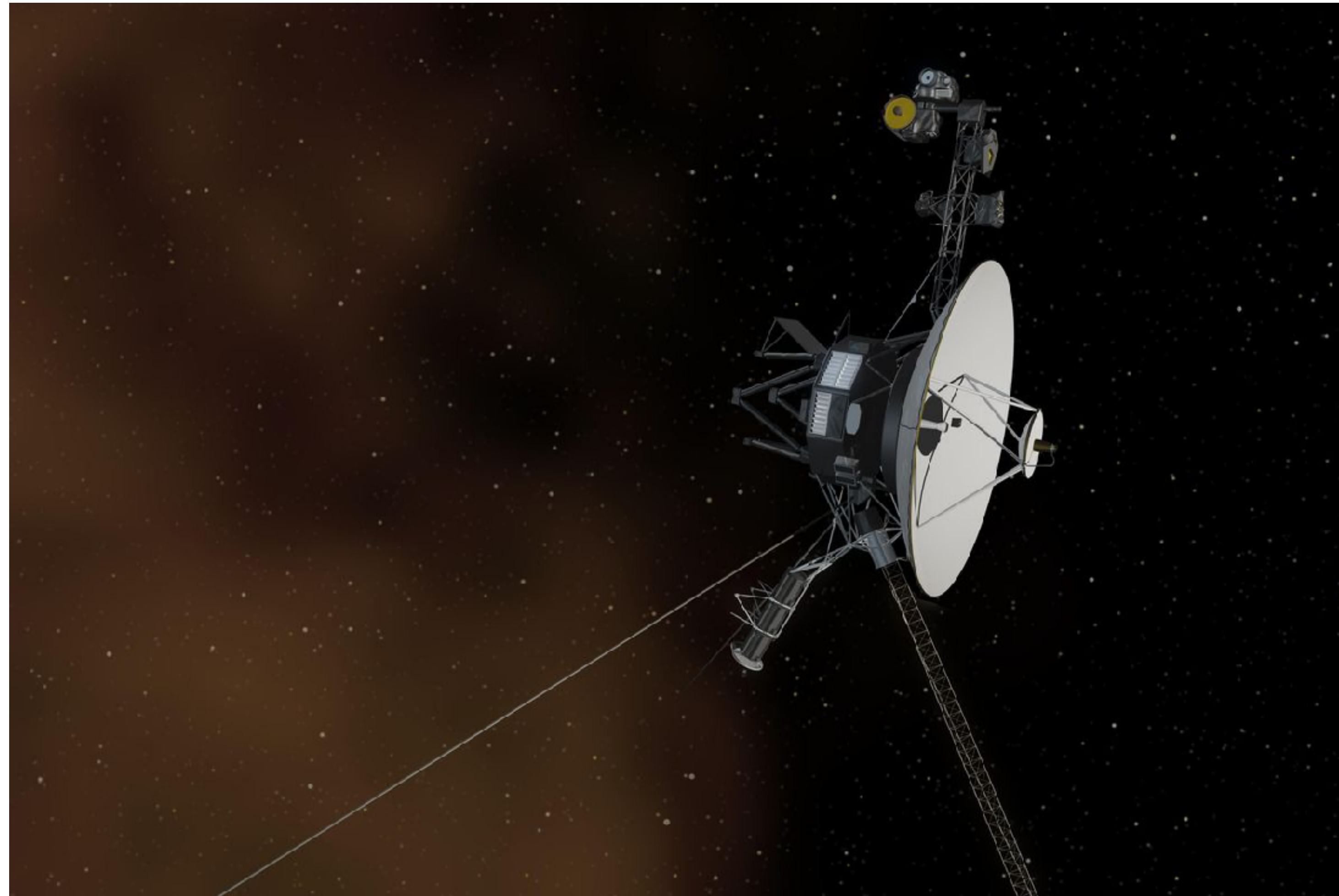
- ω_p is exactly the frequency we have defined earlier



- plasma oscillations and Debye shielding are examples of collective behaviour that is typical in plasma
- in a plasma collective behaviour must dominate over individual collisions: $\omega_p \gg \nu_c$ (ν_c is the electron-neutral collision frequency) or $\omega_p \tau \gg 1$
- number of particles in Debye sphere needs to be much higher than one
- kinetic energy is much higher than potential energy (except within a Debye length)

- Plasmas can coexist with another states
- E.g. in the ionosphere there are regions where 99% of the gas is neutral and only 1% is ionised
- we then have a **partially ionised plasma**
- the plasma parameter is calculated with only the ionised component (and we still need to have $\Lambda \gg 1$ and that λ_D is much smaller than the typical dimensions of the system)
- Usually there is continuous exchange of charge between the particles of the neutral gas and the ones of the ionised plasma
- Medium is characterised by the ionisation degree

- single particle motion
 - simple but powerful analysis
 - enables to investigate key waves and instabilities in plasma physics
- plasma kinetic equations
 - general approach
 - can be solved using computer programs
- fluid equations
 - plasma waves and instabilities
 - interaction with electromagnetic waves



<http://www.jpl.nasa.gov/interstellarvoyager/>