Plasma Physics and Technology electrostatic electron plasma waves

TÉCNICO LISBOA

Mestrado Integrado em Engenharia Física Tecnológica

Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico

How to study plasmas

- single particle motion
  - simple but powerful analysis
  - enables to investigate key waves and instabilities in plasma physics

- plasma kinetic equations
  - general approach
  - can be solved using computer programs

- **fluid equations**
  - plasma waves and instabilities
  - interaction with electromagnetic waves

why study plasma waves is interesting?

- propagate information

- may become unstable: waves may grow or damp
    - amplify electromagnetic radiation to unprecedented intensities
    - generate short, bright radiation bursts
    - accelerate particles
    - lead to plasma heating
    - important in plasma confinement
    - …

- allow to infer crucial information about the physics in a given system

key properties of waves in plasmas

Amplitude

- consider "small" amplitude perturbations
- suggest linearisation procedure
- example:

$$n = n_0 + n_1 + \ldots \qquad \frac{n_1}{n_0} \ll 1$$

- what is the meaning of "small"?
- e.g. take the equation of motion:

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} \gg \mathbf{v} \cdot \nabla \mathbf{v}$$

Frequency and wavenumber

- frequency ω
- wavenumber $\mathbf{k}$
- suggest taking sinusoidal perturbation (Fourier transform)

$$n_1 = \bar{n}_1 \exp\left(\mathbf{k} \cdot \mathbf{r} - \omega t\right)$$

- this approach works most of the time
- some of its limits will become clearer later on, during the course.

Nicholson, pp. 133

key properties of waves in plasmas

wave description - analytical

$$\mathbf{u} = \mathbf{u}_0 \exp\left[i\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)\right]$$

- dispersion relation

  $\omega = \omega\left(\mathbf{k}\right)$

- key properties (attention to definition of $\mathbf{k}$ and ω)

  phase speed $\mathbf{v}_\phi = \dfrac{\omega}{\mathbf{k}}$

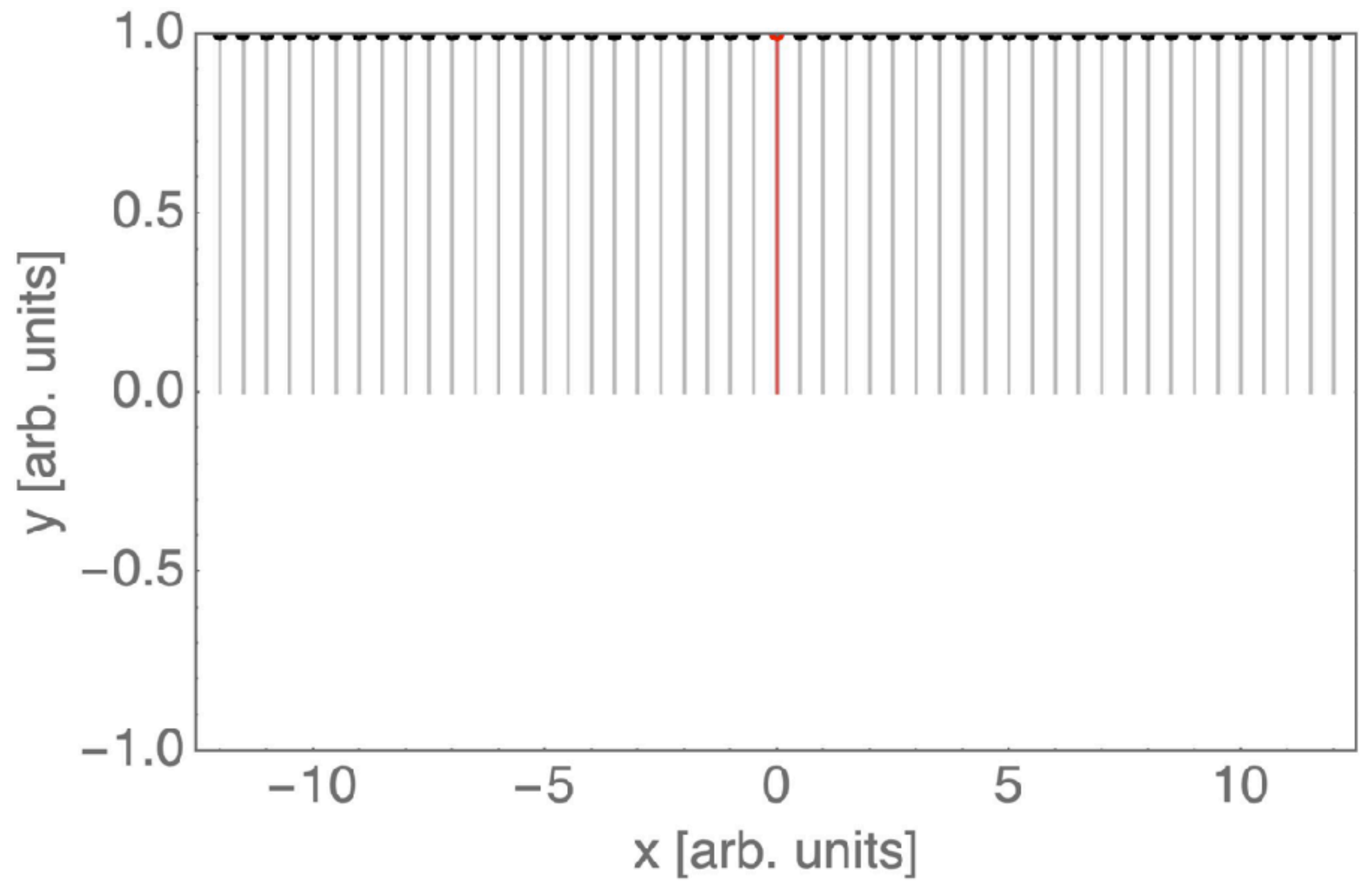
  group speed $\mathbf{v}_g = \nabla_{\mathbf{k}}\omega$

- $\mathbf{v}_\phi$ and $\mathbf{v}_g$ may/may not be equal

  in dispersive media they are not

Phase speed "mechanical" example - "plasma" oscillations

$y(t) = \cos(t)$



key properties of waves in plasmas

wave description - analytical

$$\mathbf{u} = \mathbf{u}_0 \exp\left[i\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)\right]$$

- dispersion relation

  $\omega = \omega\left(\mathbf{k}\right)$

- key properties (attention to definition of $\mathbf{k}$ and ω)

  phase speed $\mathbf{v}_\phi = \dfrac{\omega}{\mathbf{k}}$

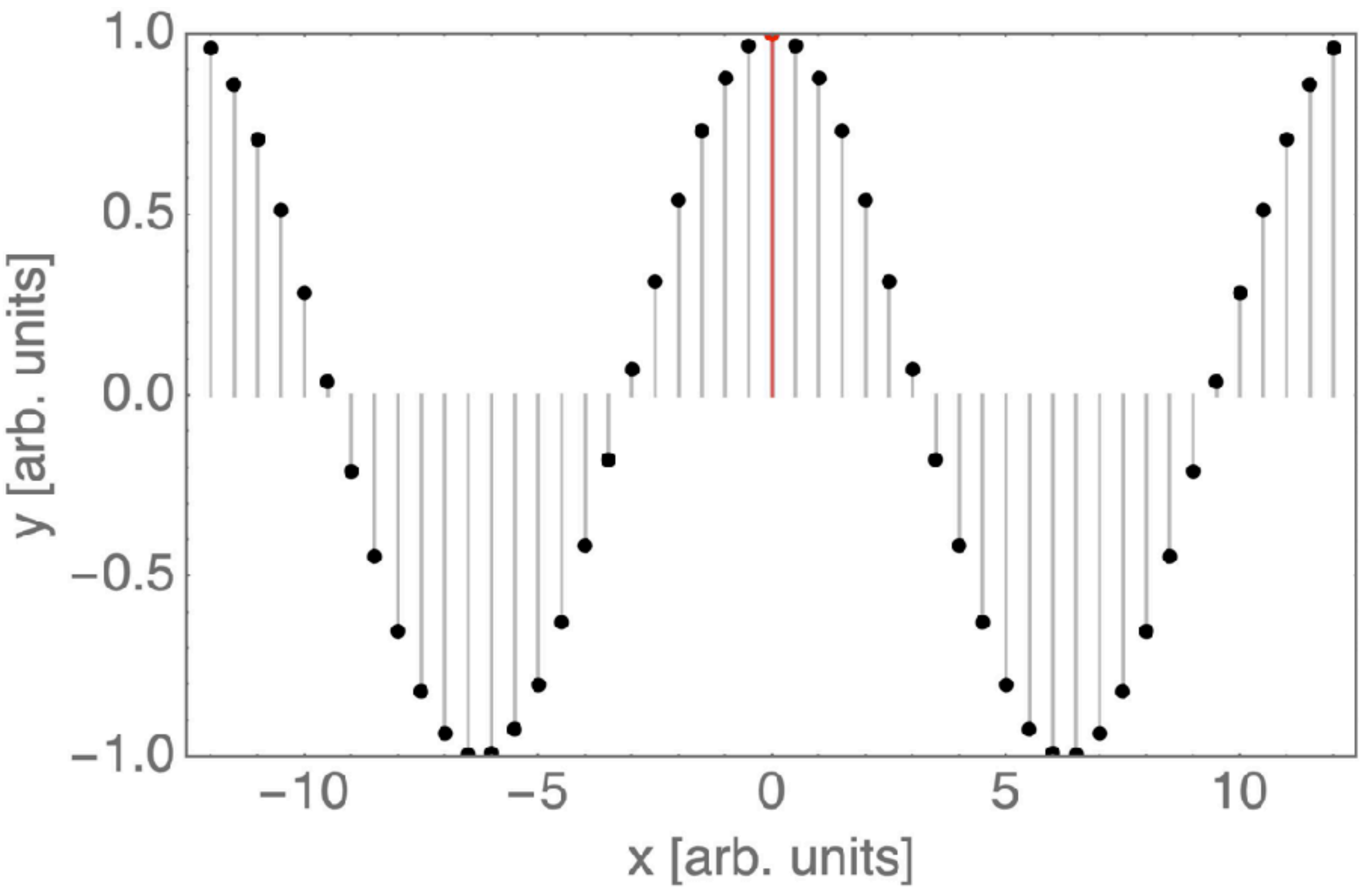
  group speed $\mathbf{v}_g = \nabla_{\mathbf{k}}\omega$

- $\mathbf{v}_\phi$ and $\mathbf{v}_g$ may/may not be equal

  in dispersive media they are not

Phase speed "mechanical" example - beat wave

$y(t) = \cos(x_0)\cos(t)$



key properties of waves in plasmas

wave description - analytical

$$\mathbf{u} = \mathbf{u}_0 \exp\left[i\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)\right]$$

- dispersion relation

  $\omega = \omega\left(\mathbf{k}\right)$
- key properties (attention to definition of $\mathbf{k}$ and ω)

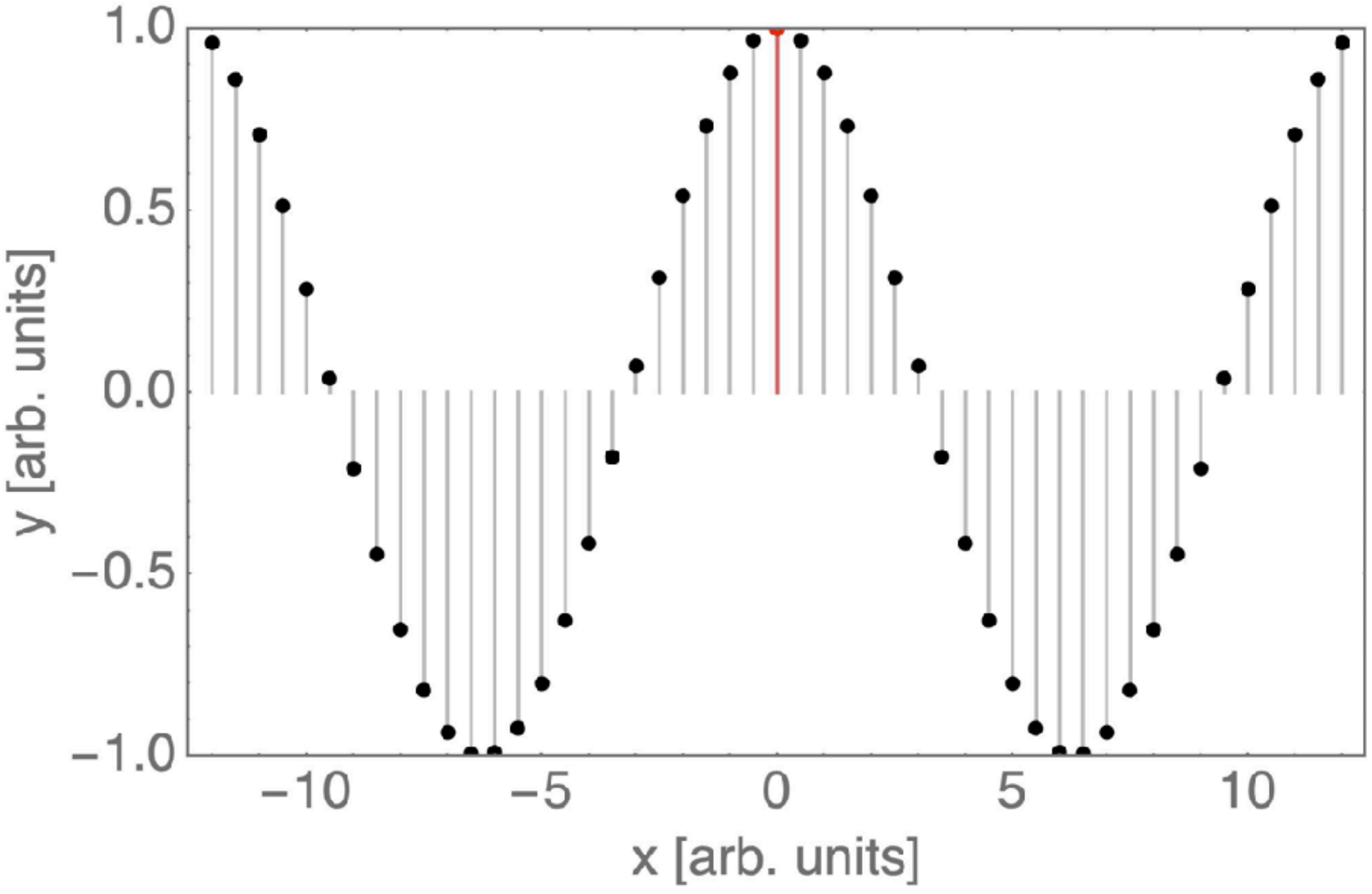
  phase speed $\mathbf{v}_\phi = \dfrac{\omega}{\mathbf{k}}$

  group speed $\mathbf{v}_g = \nabla_{\mathbf{k}}\omega$
- $\mathbf{v}_\phi$ and $\mathbf{v}_g$ may/may not be equal

  in dispersive media they are not

Phase speed “mechanical” example - travelling “plasma” oscillations

$y(t) = \cos(t - x/2)$



key properties of waves in plasmas

wave description - analytical

$$\mathbf{u} = \mathbf{u}_0 \exp\left[i\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)\right]$$

- dispersion relation

  $\omega = \omega\left(\mathbf{k}\right)$
- key properties (attention to definition of $\mathbf{k}$ and ω)

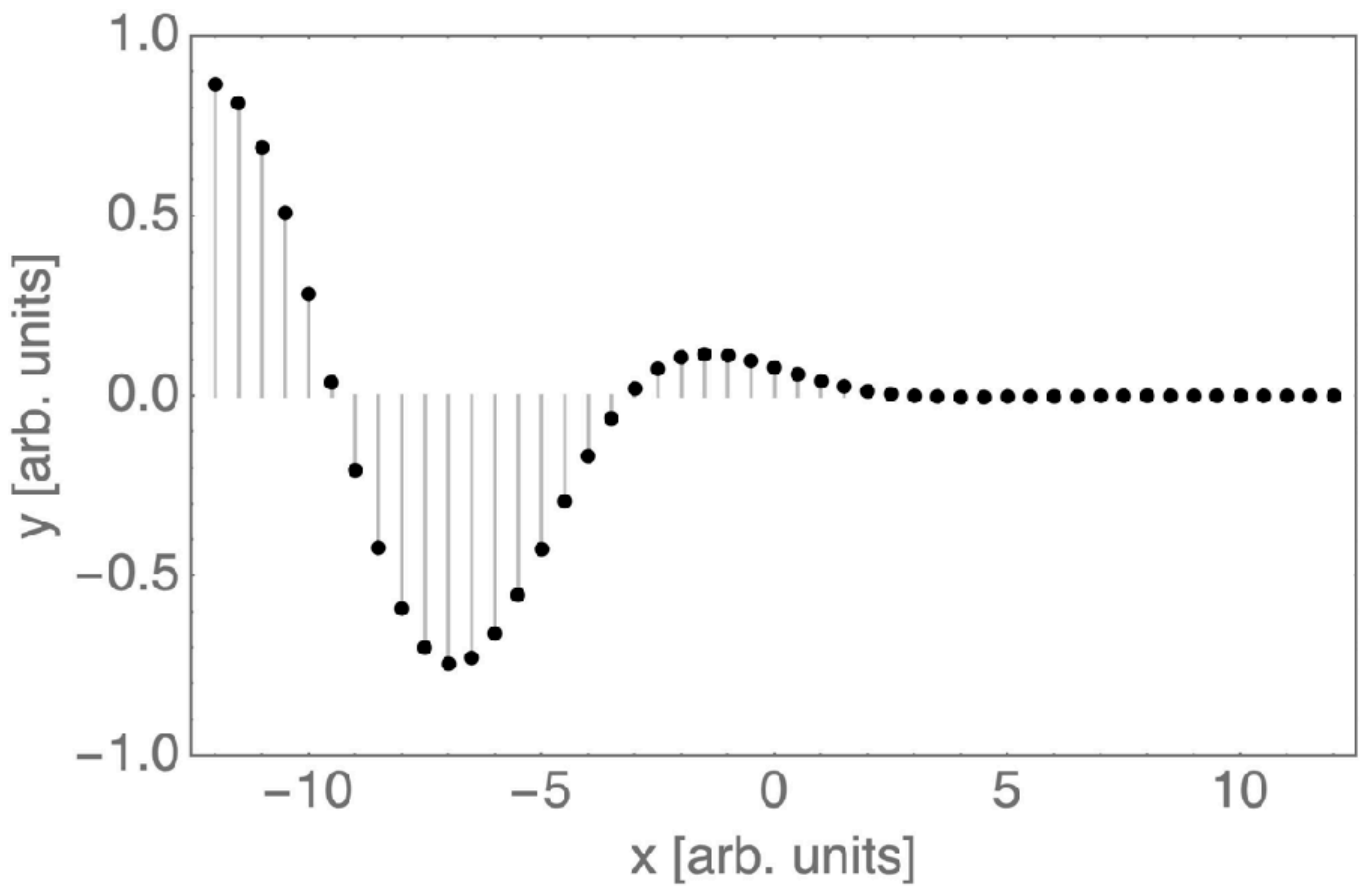
  phase speed $\mathbf{v}_\phi = \dfrac{\omega}{\mathbf{k}}$

  group speed $\mathbf{v}_g = \nabla_{\mathbf{k}}\omega$
- $\mathbf{v}_\phi$ and $\mathbf{v}_g$ may/may not be equal

  in dispersive media they are not

Group speed "mechanical" example

$y(t) = \cos(t - x/2)\exp[-(2t - x_i)^2/L_x^2]$



waves in a cold, un-magnetised infinite plasma

Assumptions

- **infinite** medium
- **no** collisions
- **immobile ions** (ions have infinite mass)
- $\dfrac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} \gg \mathbf{v} \cdot \nabla \mathbf{v}$

Maxwell's equations combined with Lorentz force

Linearised force equation (1):

$$\frac{\partial \mathbf{j}_e}{\partial t} = \frac{n_e e^2}{m_e}\mathbf{E}$$

Faraday's law + Ampere's law (2)

$$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{j}}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Combine (1) and (2)

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} + \frac{\omega_p^2}{c^2}\mathbf{E} = 0$$

Dawson, 220

electrostatic electron waves in a cold plasma

Assumptions

- **electrostatic** (also called longitudinal) plasma waves

$$\mathbf{k} \times \mathbf{E} = 0$$

- Gauss's law - **finite** plasma density fluctuations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \neq 0$$

- These waves are purely electrostatic

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = 0$$

Derivation and properties

$$\cancel{\nabla \times \nabla} \times \mathbf{E} = -\frac{\omega_p^2}{c^2}\mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

Fourier analysing gives:

$$\omega^2 = \omega_p^2$$

Phase velocity:

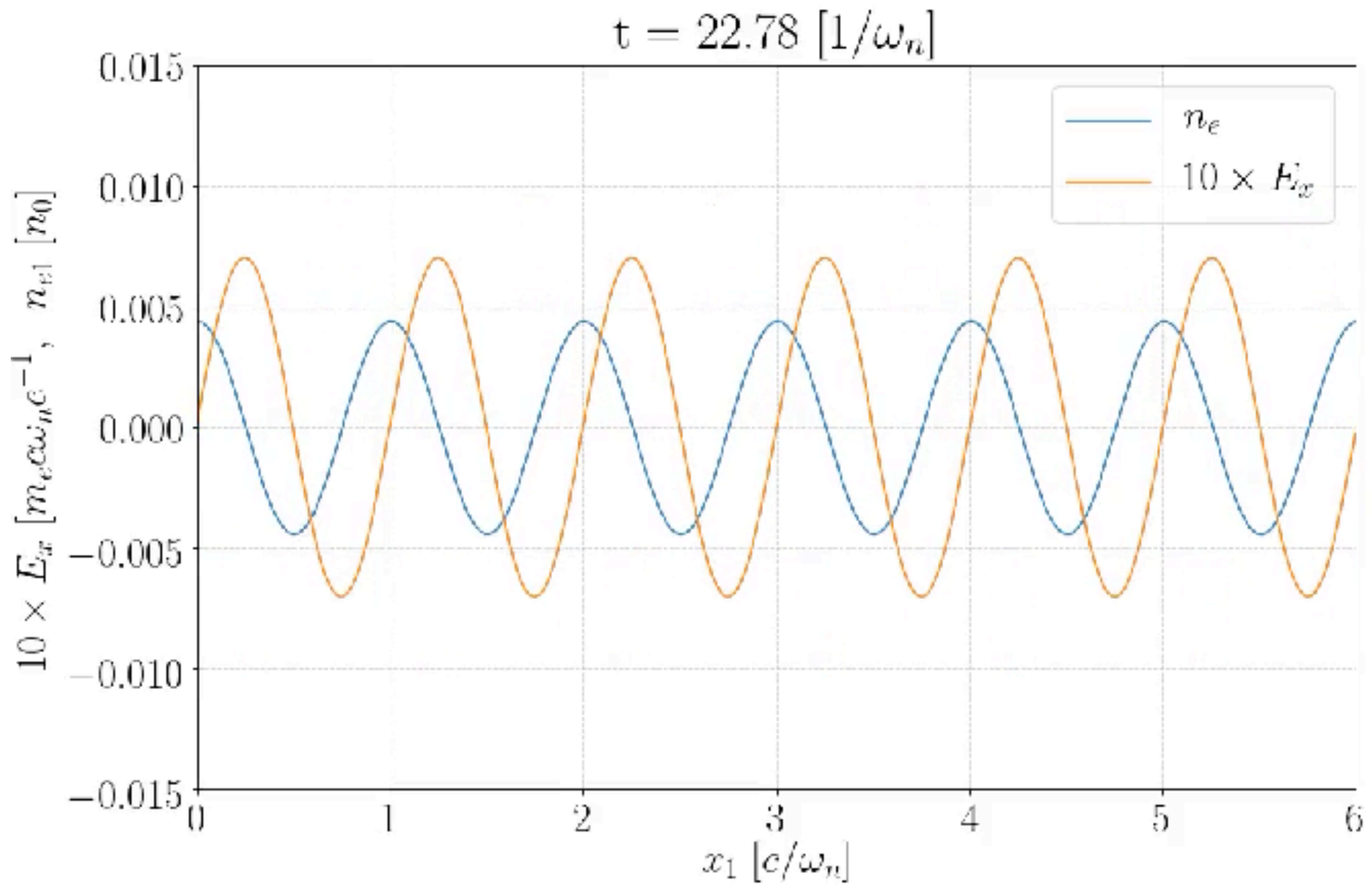
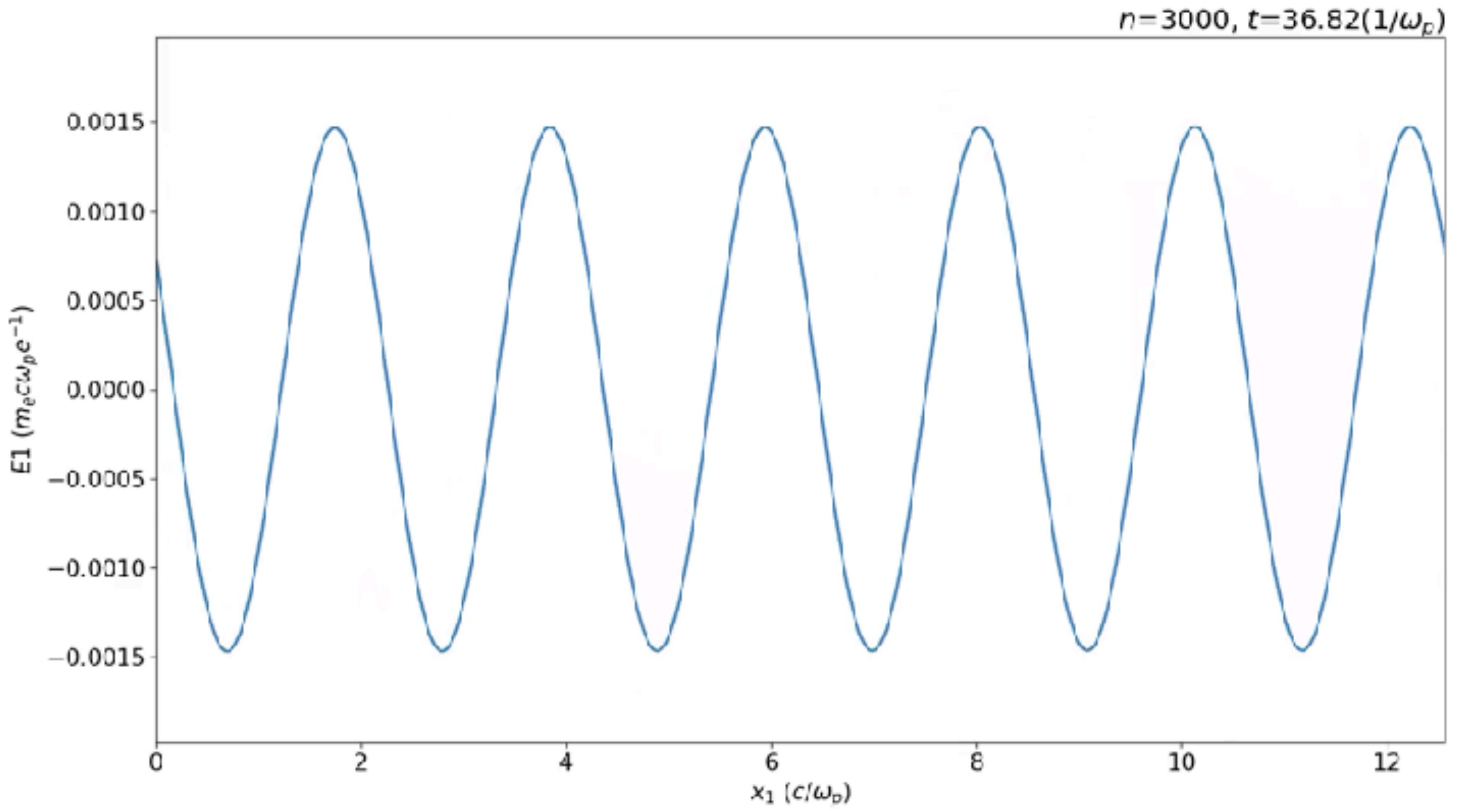
$$\mathbf{v}_\phi = \frac{\omega_p}{k^2}\mathbf{k}$$

Group velocity:

$$\mathbf{v}_g = \mathbf{0}$$

Dawson, 224

waves in a cold, un-magnetised infinite plasma

No phase velocity	Finite phase velocity
	**Variable amplitude…** 

electrostatic electron plasma waves in warm plasma

Assumptions

- keep all previous assumptions except

$$\nabla P \neq 0$$

- adiabatic transformation (electrons travel only a fraction of the wavelength in a plasma period)

$$\frac{v_e}{\omega} \ll \lambda$$

- collision frequency is small (pressure fluctuations are not transmitted to the other directions)

$$\nu_c \ll \omega$$

- 1D adiabatic compressions:

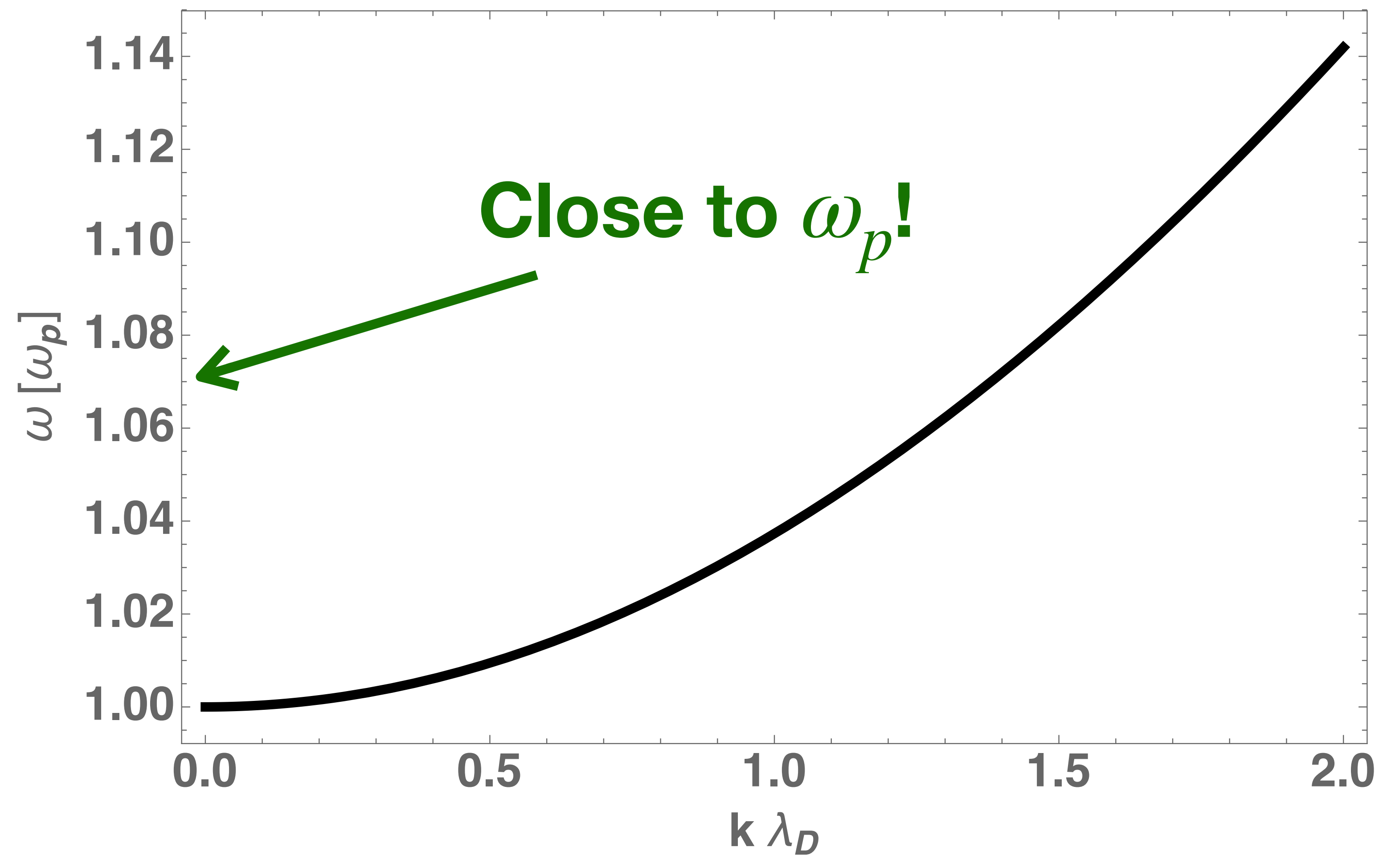
$$Pn^{-3} = \text{constant}$$

Dispersion relation for the Langmuir waves

- calculations…

$$\omega^2 = \omega_p^2 + 3k^2 v_{\text{th,e}}^2$$

$$v_{\text{th,e}}^2 = \frac{k_B T_e}{m_e}$$



Nicholson, 132-136 (partial overlap with Dawson)

electrostatic electron plasma waves in warm plasma

Assumptions

- keep all previous assumptions except

$$\nabla P \neq 0$$

- adiabatic transformation (electrons travel only a fraction of the wavelength in a plasma period)

$$\frac{v_e}{\omega} \ll \lambda$$

- collision frequency is small (pressure fluctuations are not transmitted to the other directions)

$$\nu_c \ll \omega$$

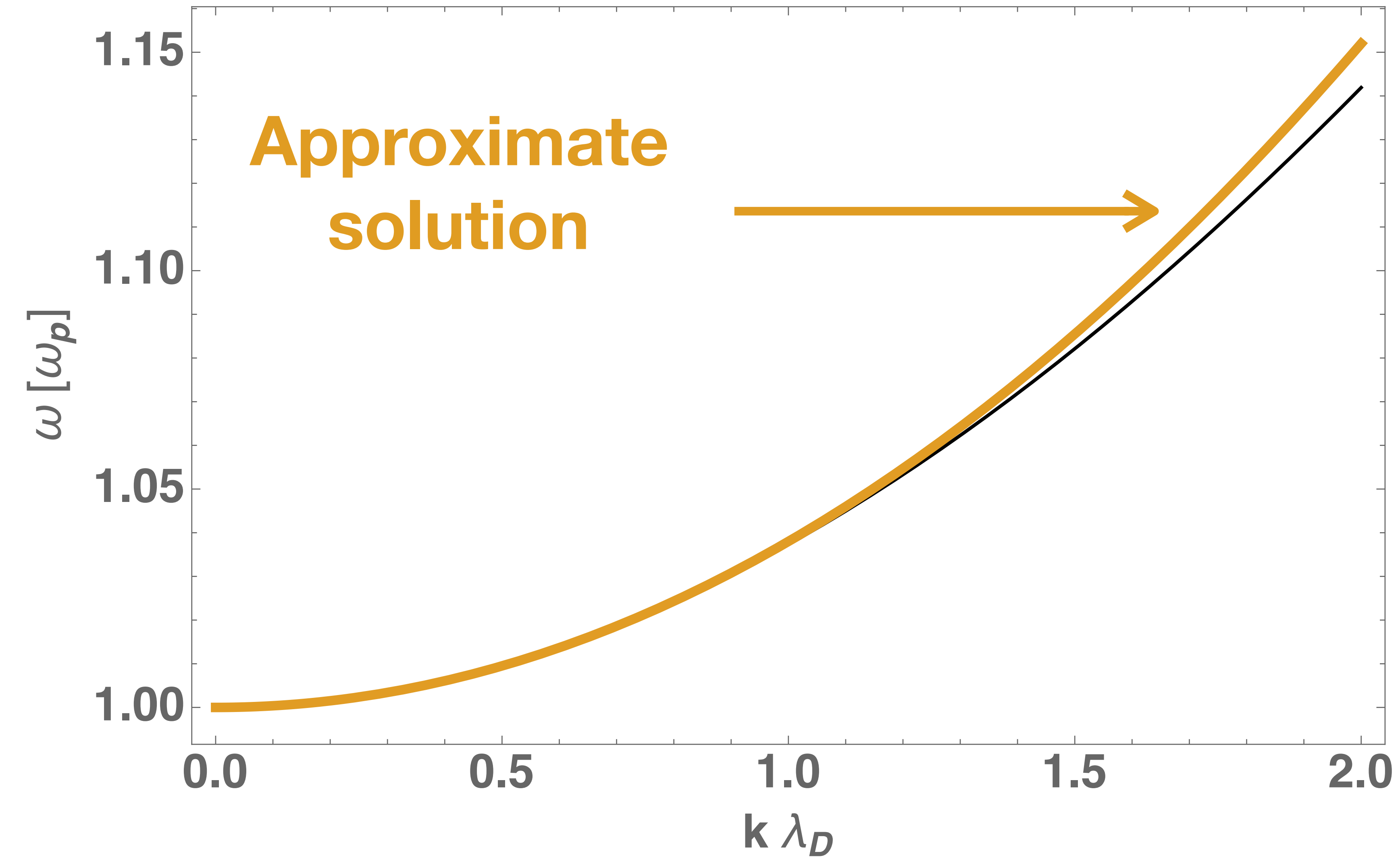
- 1D adiabatic compressions:

$$Pn^{-3} = \text{constant}$$

Approximate solution

- when $k\lambda_d \ll 1$

$$\omega = \omega_p + \frac{3}{2}k^2\lambda_d^2$$



Nicholson, 132-136 (partial overlap with Dawson)

electrostatic electron plasma waves in warm plasma

Properties

- group velocity

$$v_g = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{3v_{\mathrm{th,e}}^2}{v_\phi}$$

- validity of adiabatic hypothesis:

$$v_{\mathrm{th,e}} \ll \frac{\omega}{k} \simeq \frac{\omega_p}{k}$$

- for future thoughts: what happens when the adiabatic hypothesis fails?

Numerical check - key simulation parameters

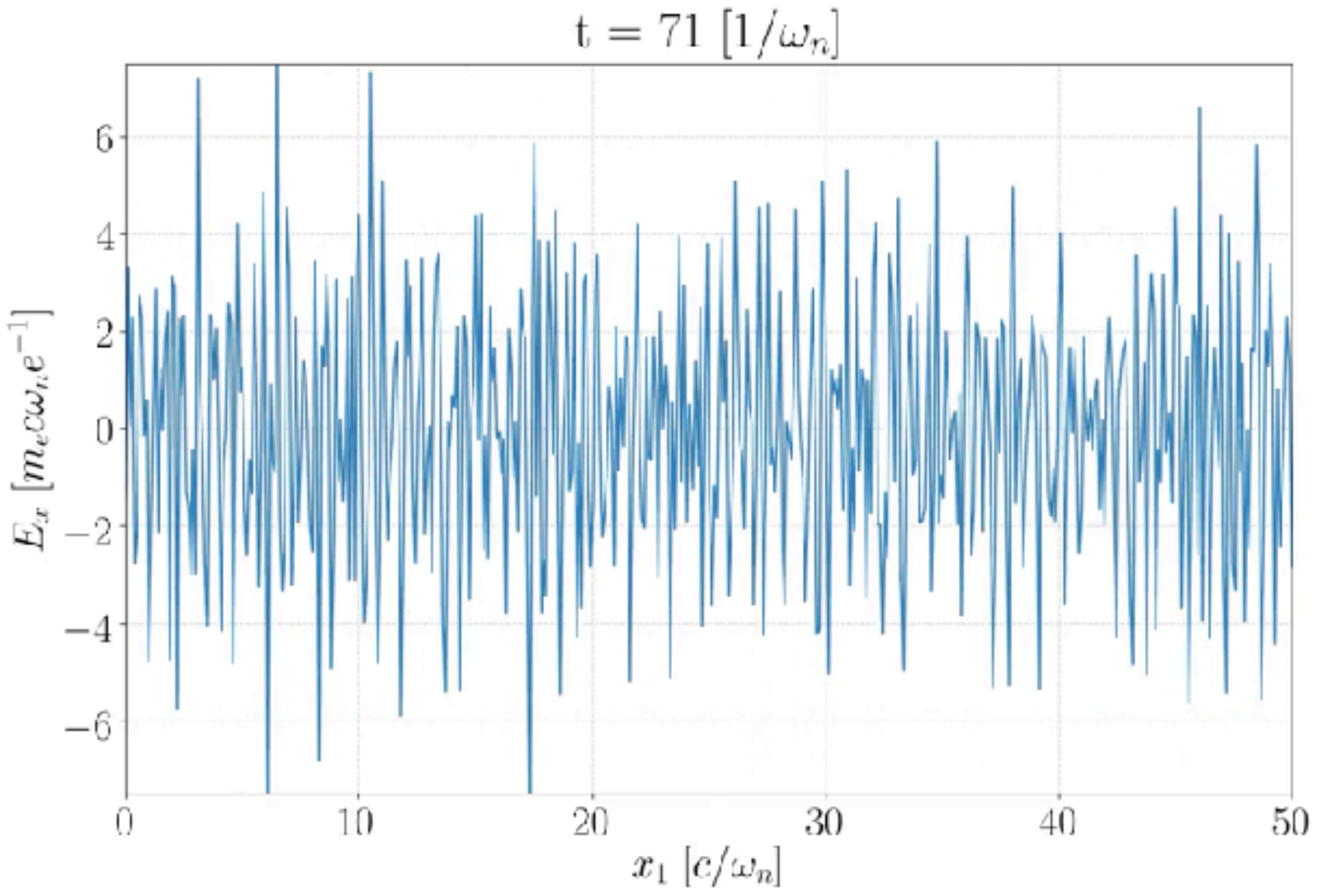
- 1D geometry
- immobile ions
- electron temperature $v_{\mathrm{th,e}} = 0.05c$ (non-relativistic!)
- box length: $100c/\omega_p$
- number of cells: 1000

Nicholson, 132-136 (partial overlap with Dawson)

electrostatic electron plasma waves in warm plasma

Result - space-time electric field evolution

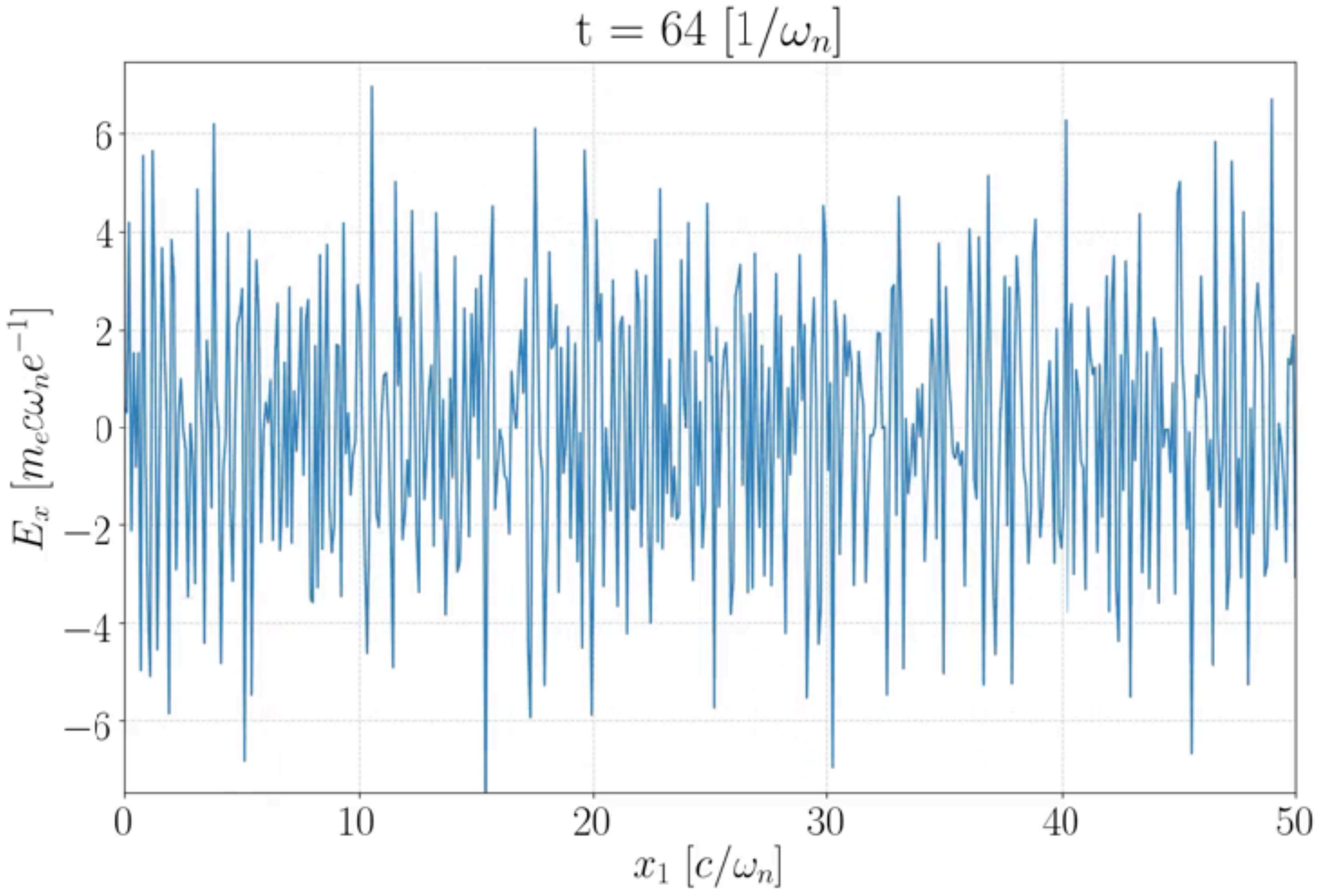
evidence of electron plasma waves?



electrostatic electron plasma waves in warm plasma

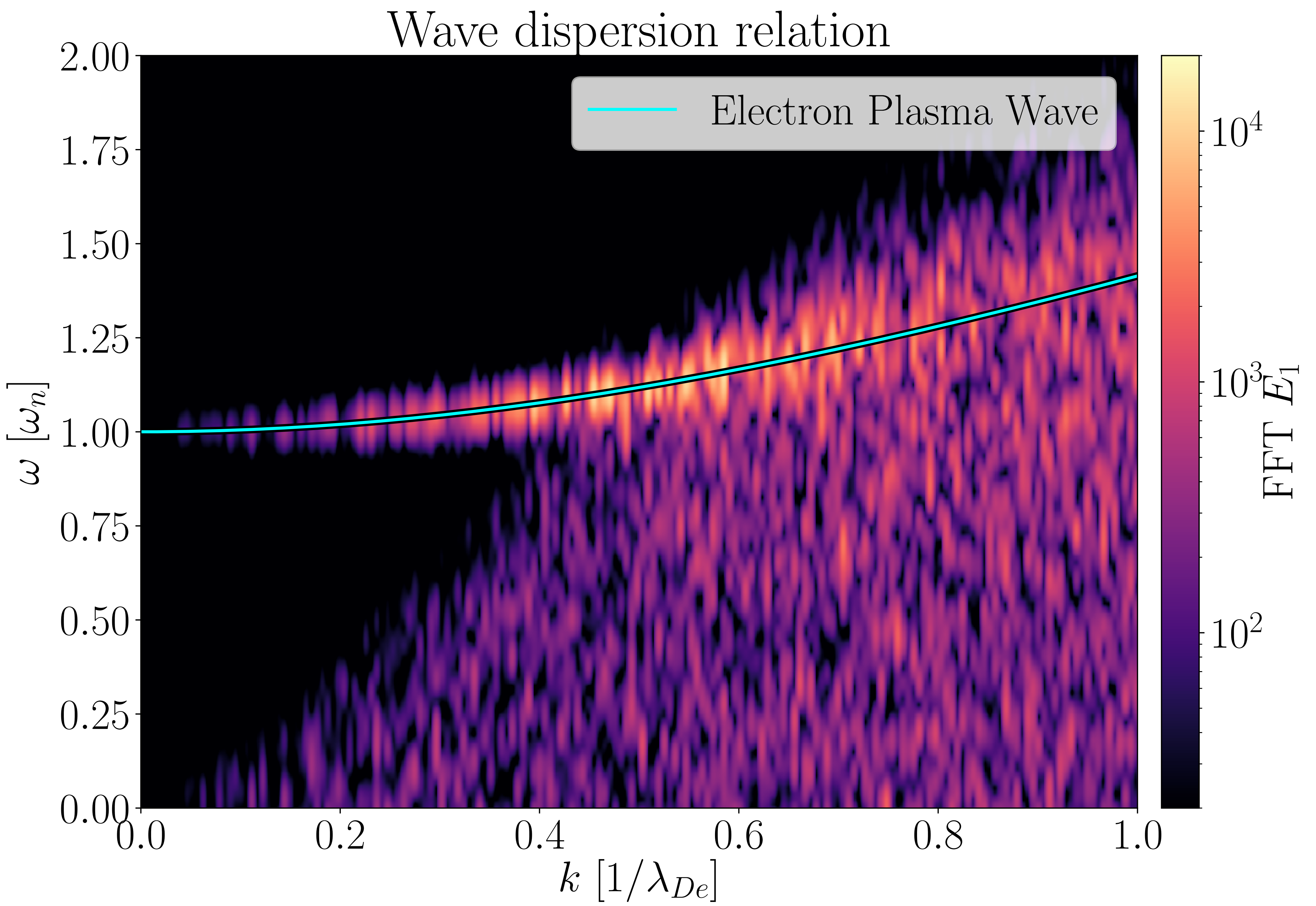
Result - space-time electric field evolution

evidence of electron plasma waves?



Result - Fourier transform

Dispersion relation



dielectric constant

macroscopic plasma response

- Gauss's law (1D)

$$ikE = \frac{\rho}{\epsilon_0} \Leftrightarrow ik\left(E - \frac{\rho}{ik\epsilon_0}\right) \equiv ik\epsilon E = 0$$

- Note for when the electric field exists in more than one direction we wish to write:

$$D\mathbf{E} = 0$$

- dispersion relation comes from:

$$\det(D) = 0$$

Cold and warm (Langmuir) plasma waves

- Cold electron plasma waves:

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

dispersion relation:

$$\epsilon(\omega) = 0 \Leftrightarrow \omega = \pm\,\omega_p$$

- Langmuir waves

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - 3k^2 v_{\text{th},e}}$$

dispersion relation:

$$\epsilon(\omega) = 0 \Leftrightarrow \omega = \pm\left(\omega_p^2 + 3k^2 v_{\text{th,e}}^2\right)^{1/2}$$

Nicholson, 136-137