Plasma Physics and Technology
Diffusion in weakly ionised plasmas

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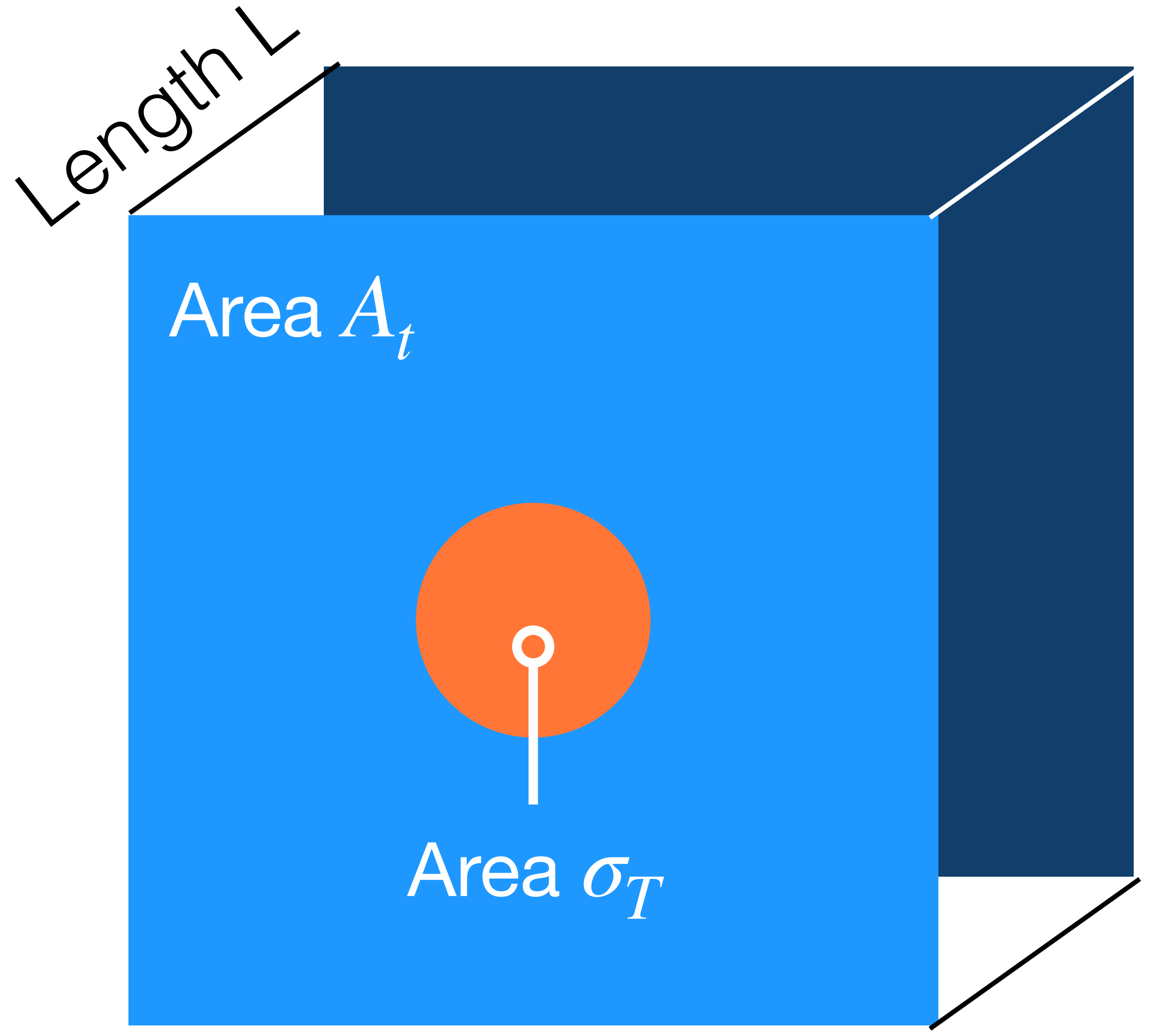
Collisions - basic concepts

Single scattering center

Box (volume LA_t) with a scattering center



Probability of test particle making a **single collision** per unit length

$$P = \sigma_T / (A_t L)$$

Dawson, pp 22-23

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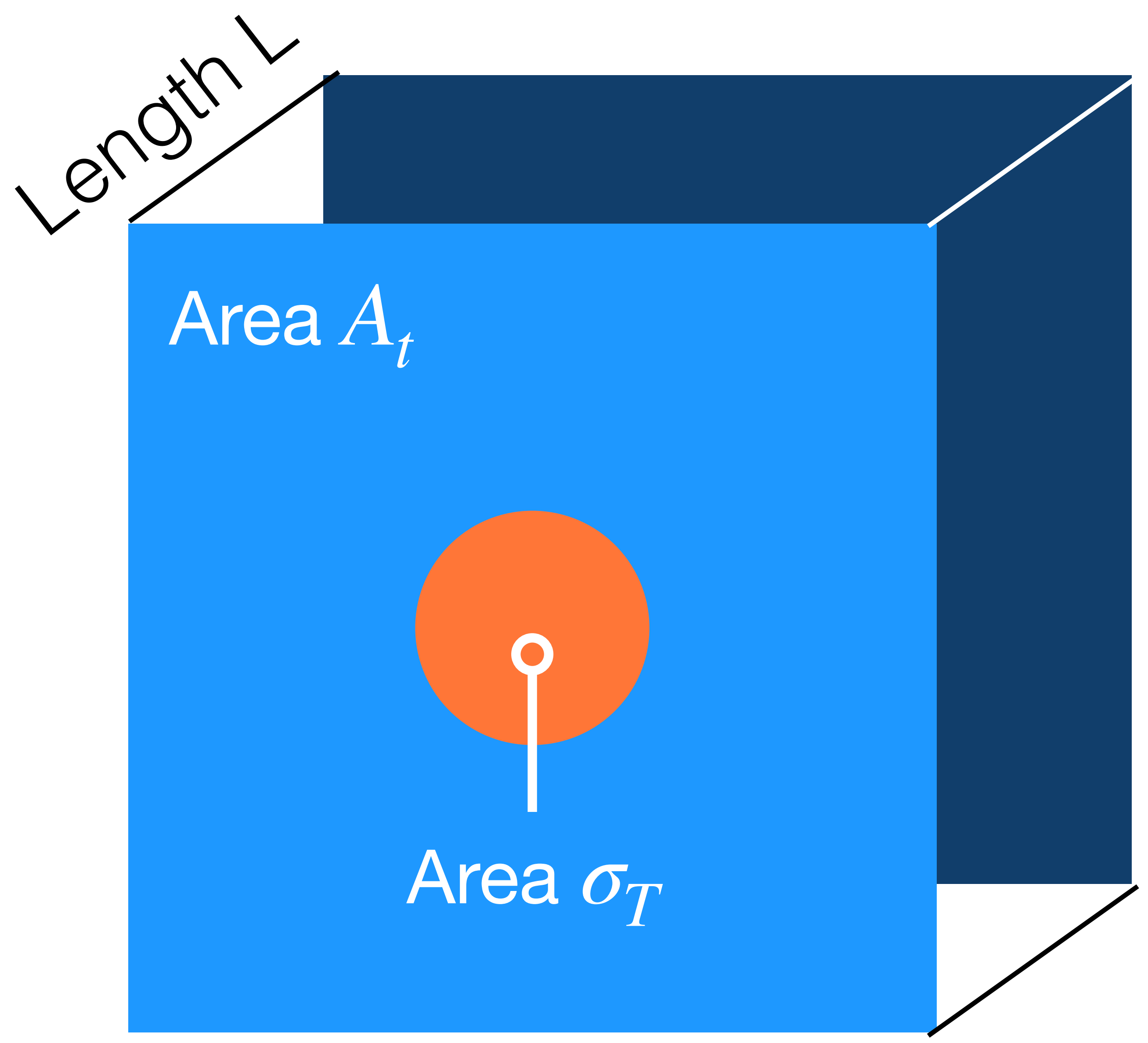

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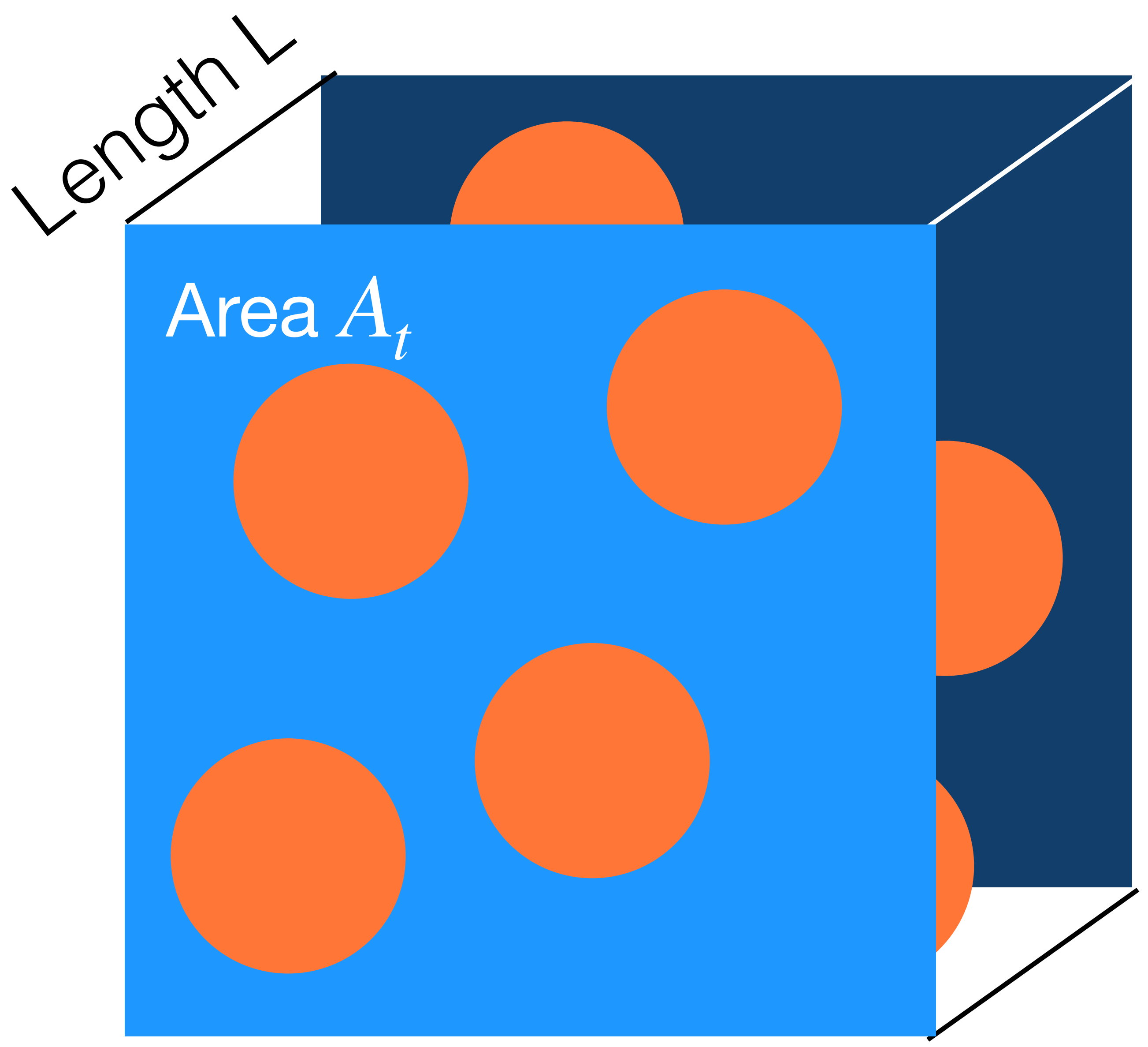

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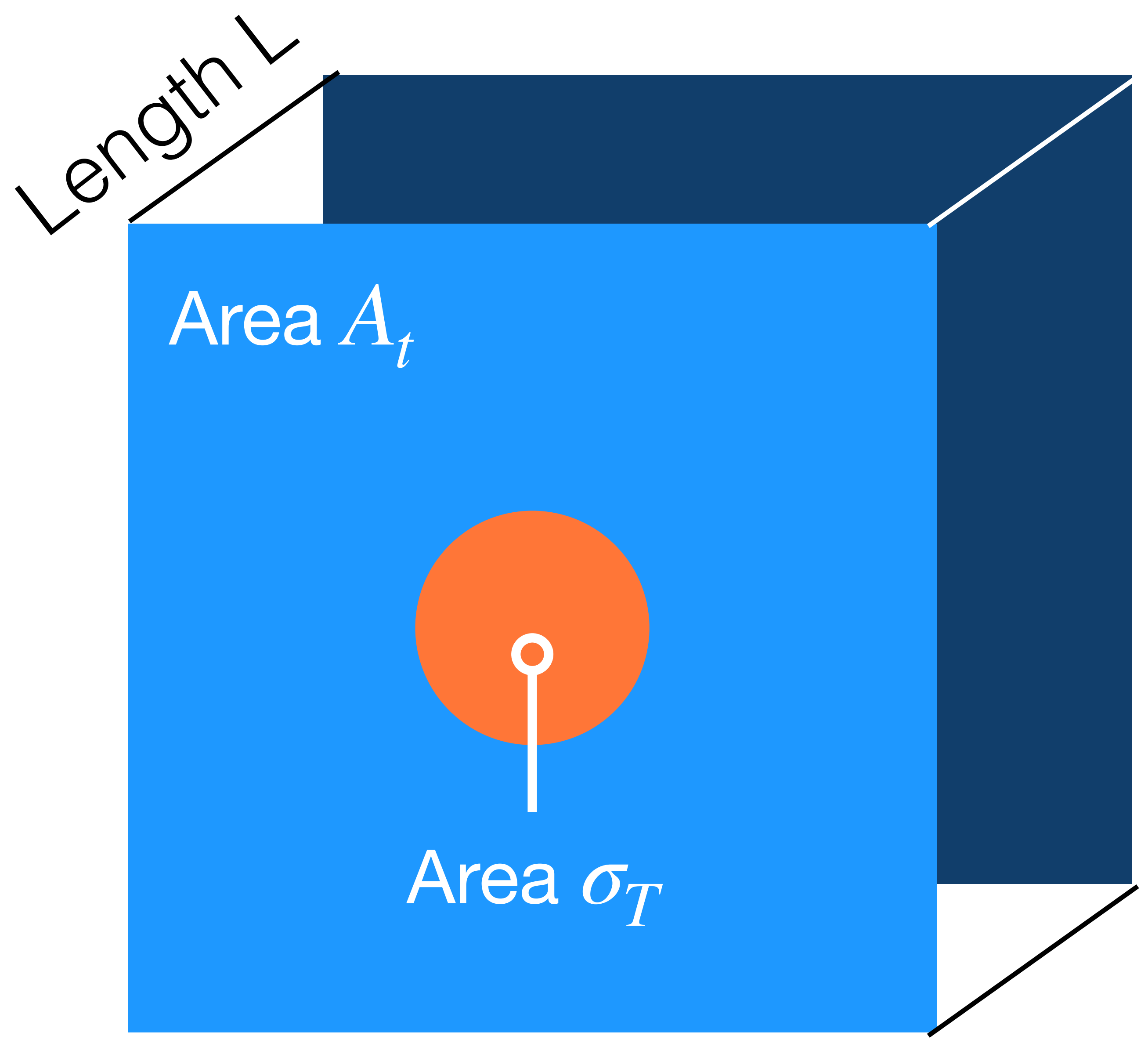

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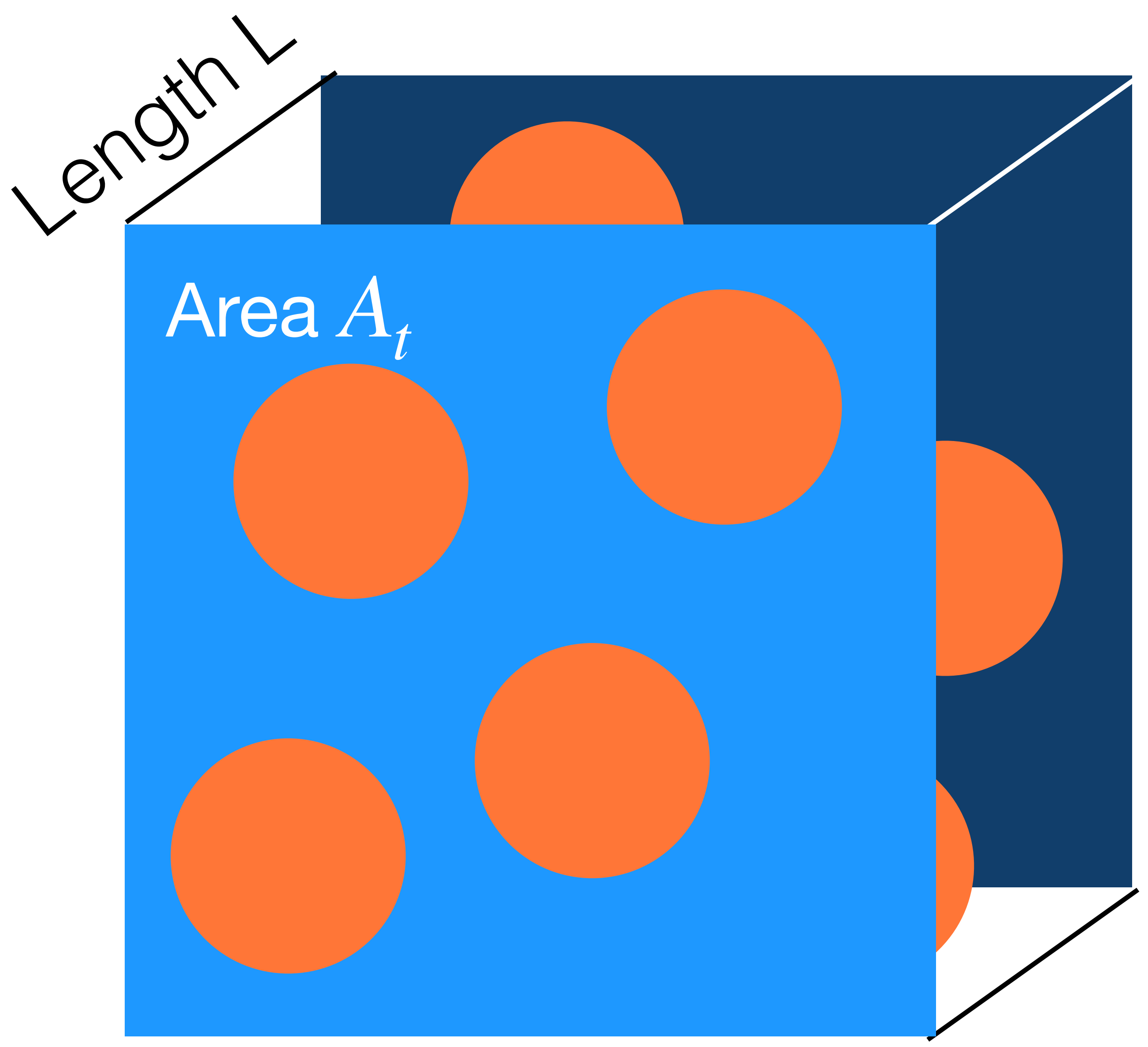

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$$P = \sigma_T n$$

Cross section

The total cross section in this model is corresponds to σ_T

Dawson, pp 22-23

Collisions - basic concepts

Mean free path λ (thin box $L = dx$)

Number of scattering centres:

$nA_t dx$

Fraction of the thin box blocked by scattering centres:

$n dx \sigma_T$

Particle flux after slab

$$I_f = I_i(1 - n\sigma_T dx) \Rightarrow \frac{dI}{dx} = -In\sigma_T$$

Typical length:

$\lambda = 1/(n\sigma_T)$

Chen, pp 157

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Collision frequency

Average time between collisions

$\Delta T = 1/(vn\sigma_T)$

Collisions per unit time

$\nu = vn\sigma_T$

Note:

v is the relative velocity between particle and scattering centres

Or v is the velocity in the frame where the scattering center is fixed.

Dawson, pp 22-23

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Average collision frequency

The total cross section may be (usually is) function of v.

Average collision frequency:

$$n\langle\sigma_T v\rangle = \int f(\mathbf{v})\sigma_T(\mathbf{v} - \mathbf{v}_0)\,|\,\mathbf{v} - \mathbf{v}_0\,|\,d\mathbf{v}$$

- Velocity of scattering centres $f(\mathbf{v})$
- Test particle velocities $\mathbf{v}_0$

Dawson, p 23

Collisions - differential cross section

- Consider collision experiment where detector measures number of particles per unit time, $N_\Omega d\Omega$, scattered into an element of solid angle $d\Omega = \sin(\theta)d\theta d\varphi$ in the direction (θ, φ)

- This number is proportional to the incident flux of particles F_i defined as the number of particles per unit time crossing a unit area normal to the direction of incidence.

- The differential cross section is defined as:

$$\boxed{\frac{d\sigma_d}{d\Omega} = \frac{N_\Omega}{F_i}}$$

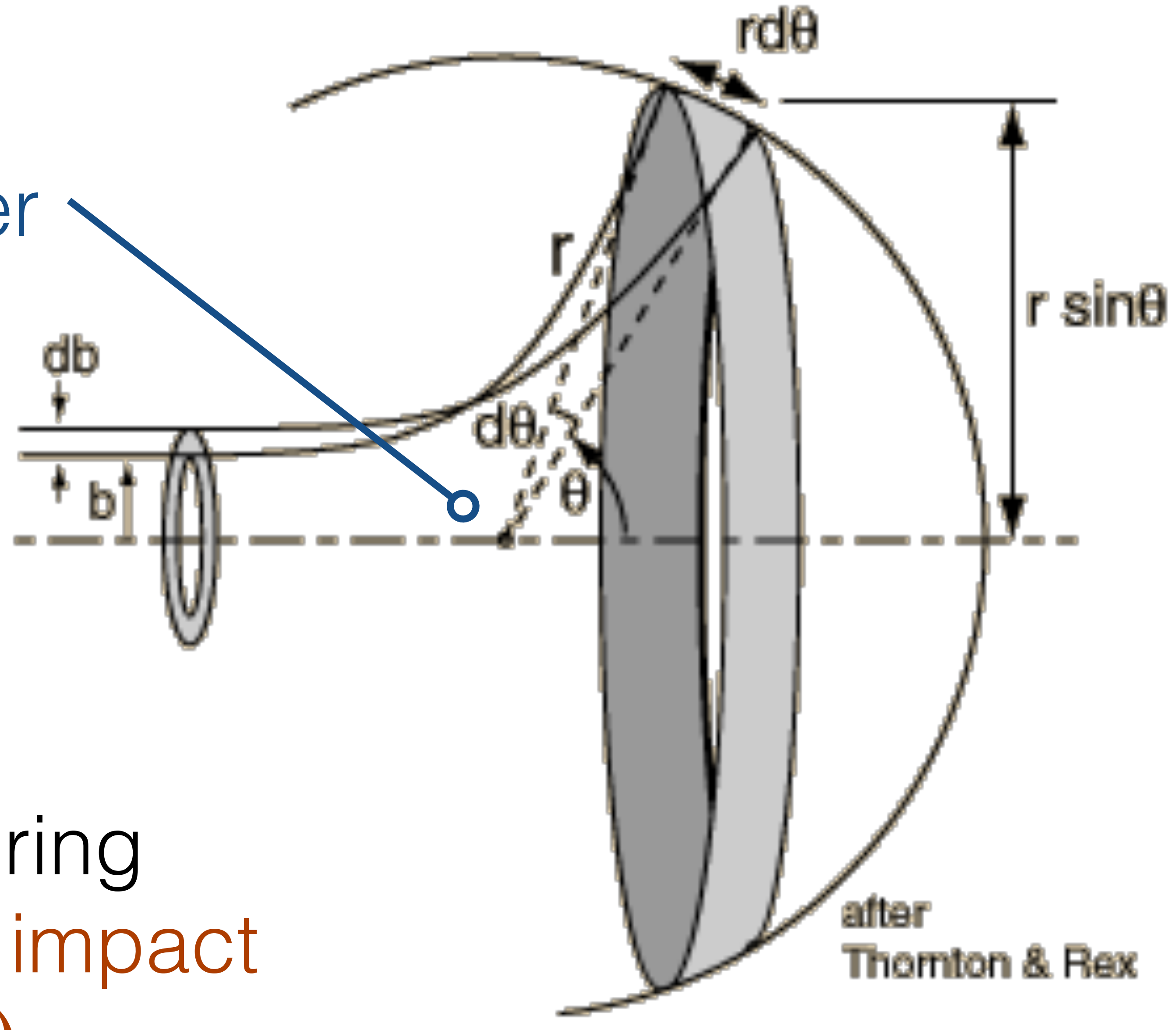
- Total cross section obtained by integrating over all solid angles:

$$\boxed{\sigma_T = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \frac{d\sigma_d}{d\Omega} \sin(\theta)}$$

Collisions - relation with total cross section

Scattering by a classical potential

Scattering potential center



Angle of scattering determined by impact parameter $b(\theta)$

Explicit formula for differential cross section

Number of particles crossing b and $b + db$ per unit time

=

Number of particles crossing θ and $\theta + d\theta$

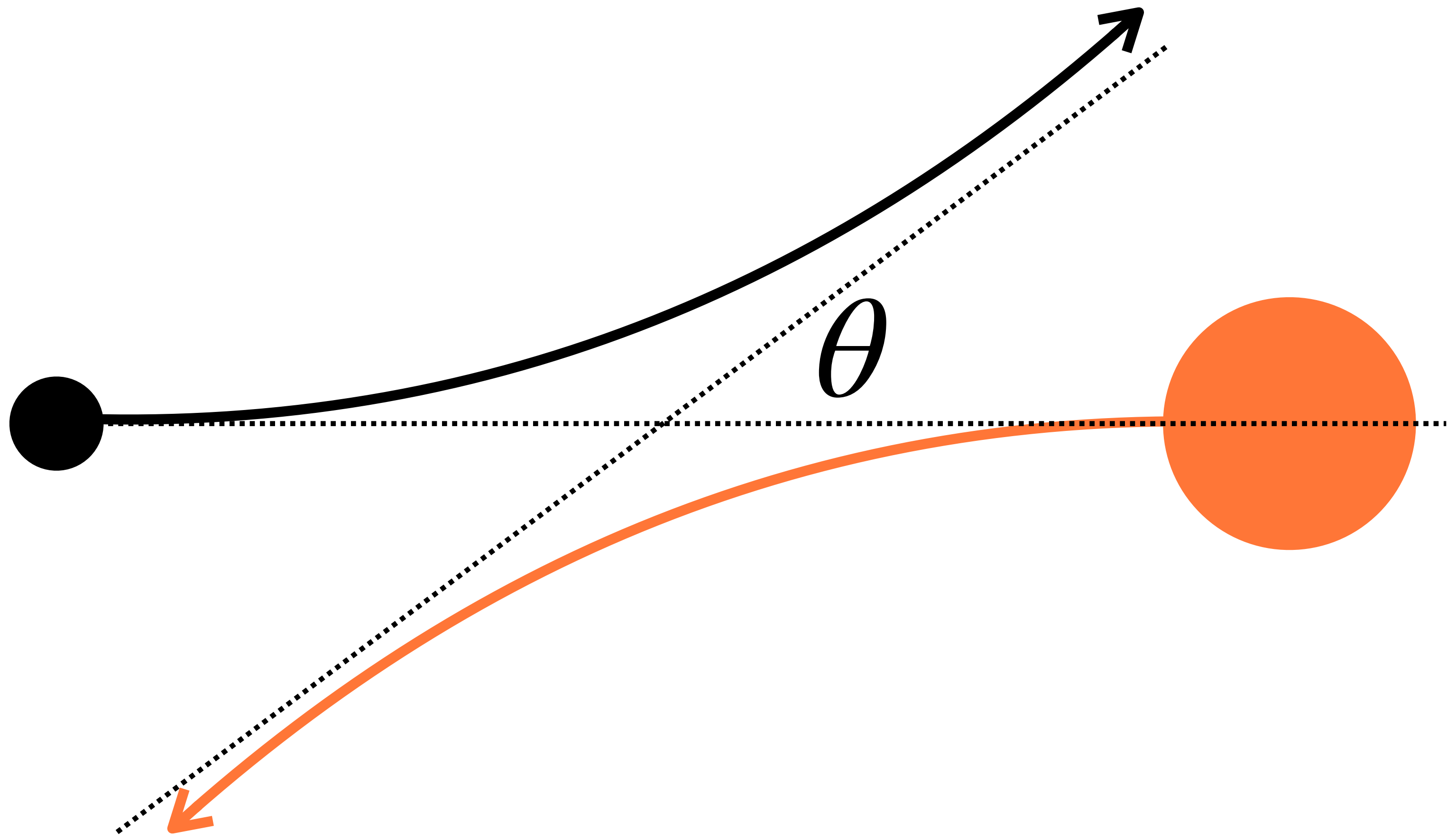
Hence:

$$N_\Omega d\Omega = N_\Omega \sin(\theta) d\theta d\varphi = F_i b db d\varphi$$

$$\boxed{\frac{d\sigma}{d\Omega} \equiv \frac{N_\Omega}{F_i} = \frac{b}{\sin(\theta)} \left(\frac{db}{d\theta} \right)}$$

Momentum transfer cross section

Collision between particle and scattering center in center of mass frame
(assume symmetry about φ):



$$p_{z,f} = p_{z,i}\left[1 - \cos(\theta)\right] \Rightarrow \Delta p_z = \left[1 - \cos(\theta)\right]$$

Average fraction of momentum transferred to the target for a single particle (assume cylindrical symmetry):

$$\frac{1}{\sigma_T}\int \left[1 - \cos(\theta)\right] b db = \frac{1}{\sigma_T}\int \left[1 - \cos(\theta)\right] \sigma(\theta) d\Omega$$

Dawson, p 72

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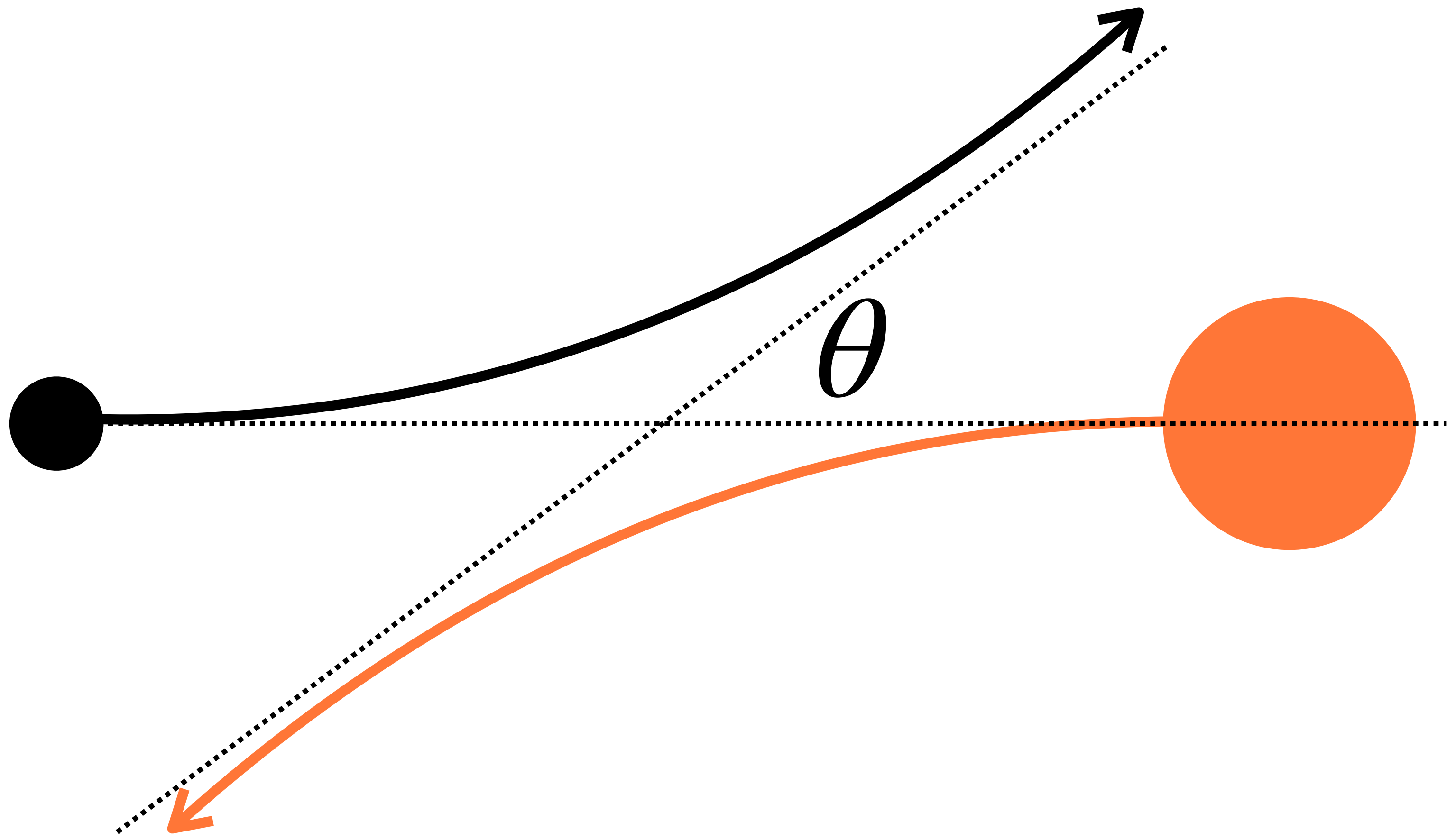


$$p_{z,f} = p_{z,i}\left[1 - \cos(\theta)\right] \Rightarrow \Delta p_z = \left[1 - \cos(\theta)\right]$$

Average fraction of momentum transferred to the target for a beam per unit time (assume cylindrical symmetry):

$$\frac{\int F_i d\sigma_d}{\sigma_T}\int\left[1 - \cos(\theta)\right]\sigma(\theta)d\Omega = F_i\int\left[1 - \cos(\theta)\right]\sigma(\theta)d\Omega$$

Dawson, p 72

Diffusion in partially ionised gases

Configuration and validity

- Electrons/ions flow through a neutral gas
- Collisions with neutrals are dominant
- Elastic collisions
- Collision frequency is independent of velocity
- What is the physical law that governs the diffusion of (charged) particles through a neutral gas?

Dawson, p 75-81

Diffusion in partially ionised gases

Generic considerations

Validity of approach

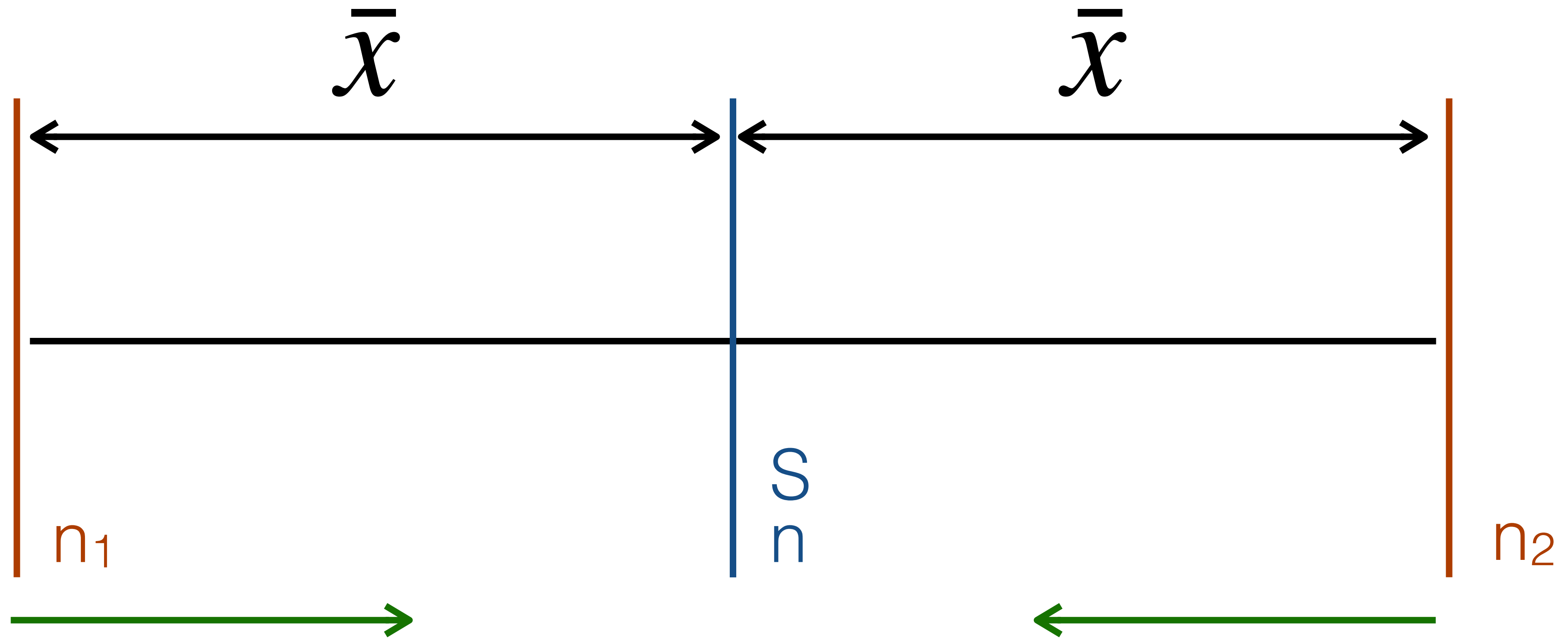
- Elastic collisions
- Collisions do not depend on velocity
- Collisions with neutrals are dominant (weakly ionised plasma)

Model (picture on the right)

- Consider thin plasma slab.
- What is the flux of electrons through S?

Physical picture 1D

What is the flux of electrons or ions through S?



$$n_1 = n - \frac{dn}{dx}\bar{x}$$

$$n_2 = n + \frac{dn}{dx}\bar{x}$$

Dawson, p 75-81

Diffusion in partially ionised gases

Diffusion coefficient

From kinetic theory the number of particles crossing a plane per unit time is (Maxwell Boltzmann):

$$\frac{1}{4}n\bar{v}$$

Thus the flux of particles through S is:

$$\Gamma = \frac{1}{4}n_1\bar{v}_1 - \frac{1}{4}n_2\bar{v}_2 = -\frac{1}{2}\frac{\partial n}{\partial x}\bar{x}\bar{v}$$

$$D = -\frac{1}{3}\lambda\bar{v} = \frac{\bar{v}^2}{3\nu_c} = \frac{k_bT}{m\nu_c}$$

Diffusion equations

Fick's Law

$$\Gamma = -D\nabla n$$

$$\nabla \cdot \Gamma = -\frac{\partial n}{\partial t}$$

Diffusion equation

$$\frac{\partial n}{\partial t} = D\nabla^2 n$$

If D varies with position

$$\Gamma = -\nabla(Dn)$$

Dawson, p 75-81

Diffusion in partially ionised gases

Mobility - generic considerations

- Elastic collisions
- Collisions with neutrals dominate
- An electric field exists
- Steady state where collisional drag compensates E field
- What is the average drift motion of an electron through the neutrals?

Dawson, p 75-81

Diffusion in partially ionised gases

Mobility

In an external electric field E electron gains energy in the direction of -E. The electron also loses a fraction of its momentum at each collision. The equation of motion is thus:

$$m\frac{d\mathbf{v}}{dt} - \alpha\nu_c' m\mathbf{v} = m\frac{d\mathbf{v}}{dt} - \nu_c m\mathbf{v} = -q\mathbf{E}$$

In a steady state

$$\mathbf{v} = -\frac{q\mathbf{E}}{\nu_c m} = -\mu\mathbf{E}$$

μ is the mobility

Relation between mobility and diffusion

The quotient D/μ obeys the Maxwell relation:

$$\frac{D}{\mu} = \frac{K_b T}{e}$$

Equilibirum: currents due to the drift cancel the currents due to diffusion.

Mobility and diffusion coefficients can be calculated by using kinetic theory by performing integrals over the distribution function.

Dawson, p 75-81

Ambipolar diffusion

Physical scenario

- Plasma in a container
- Initial ion density is equal to initial electron density
- Electron temperature is larger than ion temperature
- Because $D_e > D_i$ electrons are lost to the walls of the container
- This sets up an electric field towards the wall that slows down electron and increases the rate of ion loss.
- A steady state is formed.
- How can we quantify this phenomenon?

Dawson, p 82-86

Ambipolar diffusion

Self-generated electric field

Fluxes for ions and electrons:

$$\Gamma_{e,i} = -D_{e,i}\nabla n_{e,i} \mp \mu_{e,i} E n_{e,i}$$

Assume steady state where $\Gamma_e = \Gamma_i = \Gamma_a$ then:

$$\Gamma_a = \left(\frac{D_i\mu_e + D_e\mu_i}{\mu_i + \mu_e}\right)\nabla n = -D_a \nabla n$$

D_a (ambipolar diffusion coefficient) is also given by:

$$D_a = D_i\left(1 + \frac{T_e}{T_i}\right)$$

Self-generated electric field

$$\mathbf{E} = \frac{\nabla n}{n}\left(\frac{D_i - D_e}{\mu_i + \mu_e}\right) \simeq \frac{\nabla n}{n}\frac{k_b T_e}{e}$$

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Ambipolar diffusion

Self-generated electric field

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Validity of quasi neutrality assumption

Consider Poisson equation

$$\nabla \cdot \mathbf{E} = e\frac{n_i - n_e}{\epsilon_0} \simeq \frac{E}{L}$$

L is the characteristic length where $n_i \neq n_e$

$$E = Le\frac{n_i - n_e}{\epsilon_0} = -\frac{\nabla n}{n}\frac{k_b T_e}{e} \simeq \frac{k_b T}{eL}$$

Thus:

$$\frac{n_i - n_e}{L} \ll -\frac{\lambda_D^2}{L^2}$$

Dawson, p 82-86

Example of ambipolar diffusion in 1D

Ambipolar diffusion: density evolution

$$\frac{\partial n}{\partial t} = D_a \nabla^2 n$$

can be solved using separation of variables subject to suitable boundary conditions

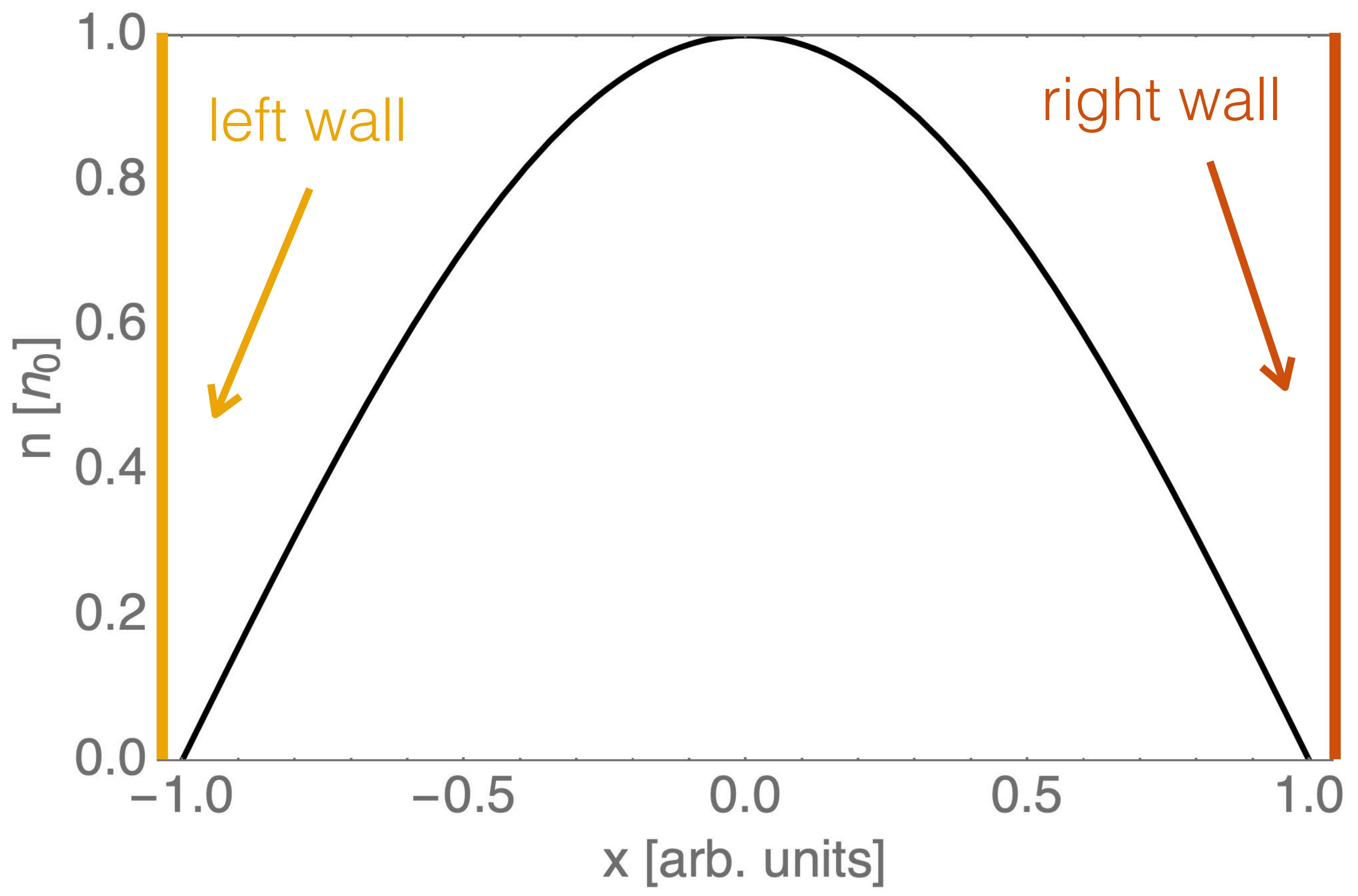
Decay of fundamental mode

Plasma slab between -L and L

$$n = n_0 \exp\left(-\frac{t}{t_d}\right) \cos\left(\frac{\pi x}{2L}\right)$$

$$t_d = 4L^2/(\pi D_a^2)$$

example



Chen, pp. 160-163

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Decay of higher order modes

see detailed calculations **Chen 162 and 163**

key point is:
fundamental mode is the slowest to diffuse

$$t_d = \left(\frac{L}{(m+1/2)\pi}\right)^2 \frac{1}{D_a}$$

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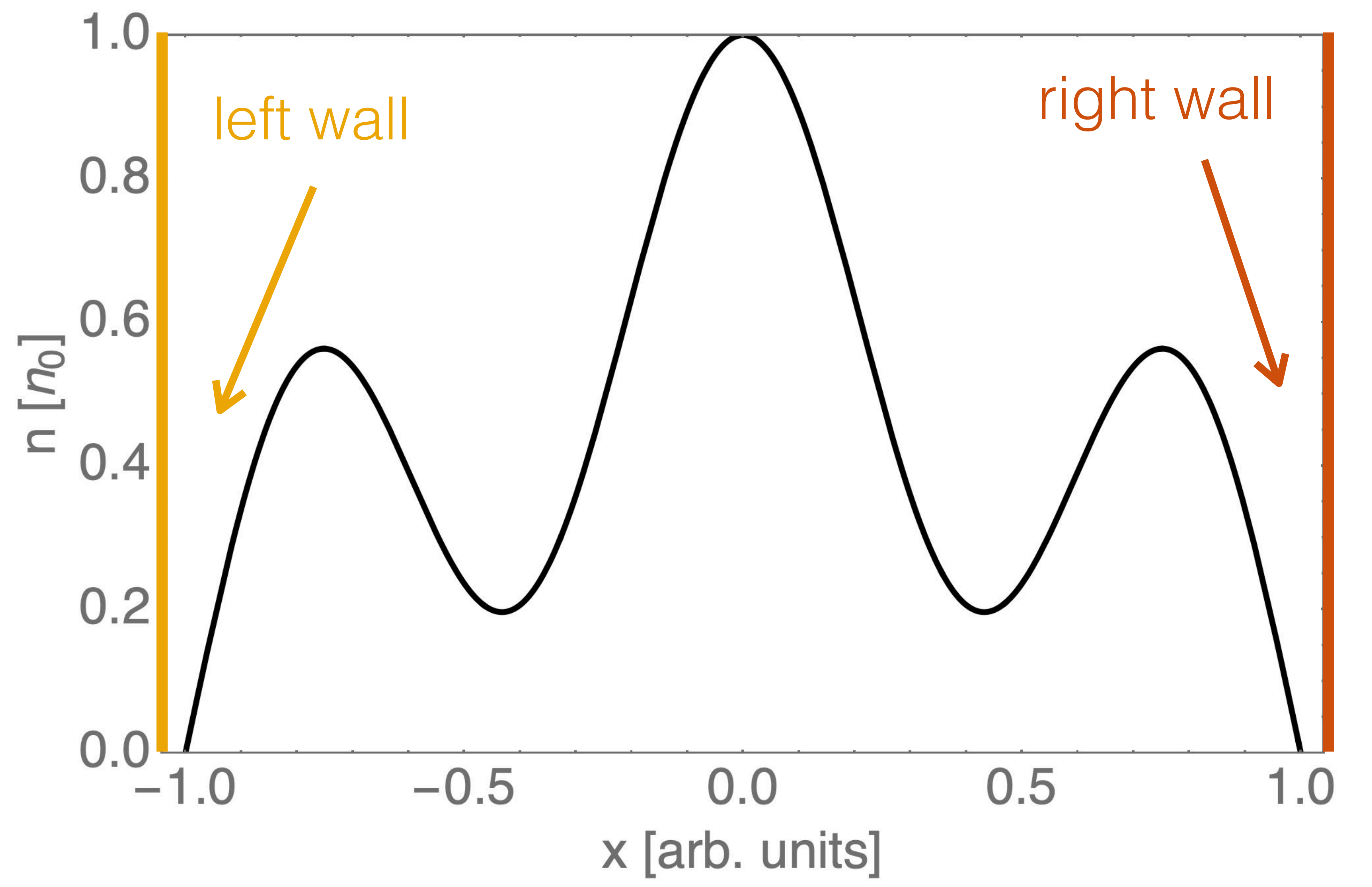

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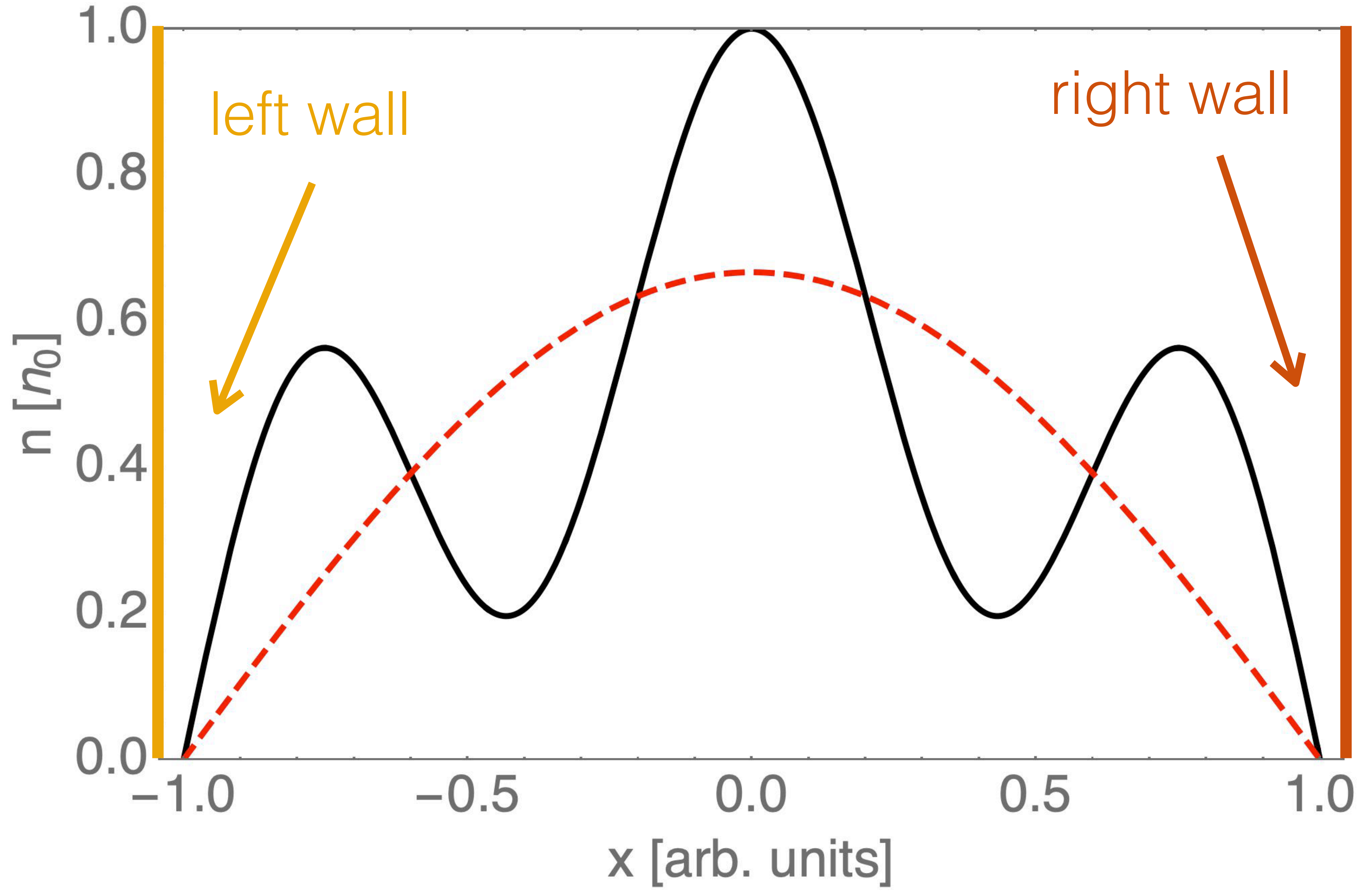

Chen, pp. 160-163

Stationary solutions: localised ionisation source

Ambipolar diffusion: density evolution

- The plasma can be maintained by injecting plasma or by ionisation processes.

- Describing the process requires addition of a source term to Continuity equation:

$$\frac{\partial n}{\partial t} - D_a \nabla^2 n = S(\mathbf{r})$$

- Constant ionisation and in steady state:

$$\nabla^2 n = -\frac{Z}{D_a} n$$

Note

- The ionisation rate Z is associated with ionisation by hot electrons at the tail of distribution

- ionisation rate is $Z = n\ \nu_i$, where ν_i is the mean ionisation frequency that can be found by integrating the ionisation cross section

Chen, pp. 165-167

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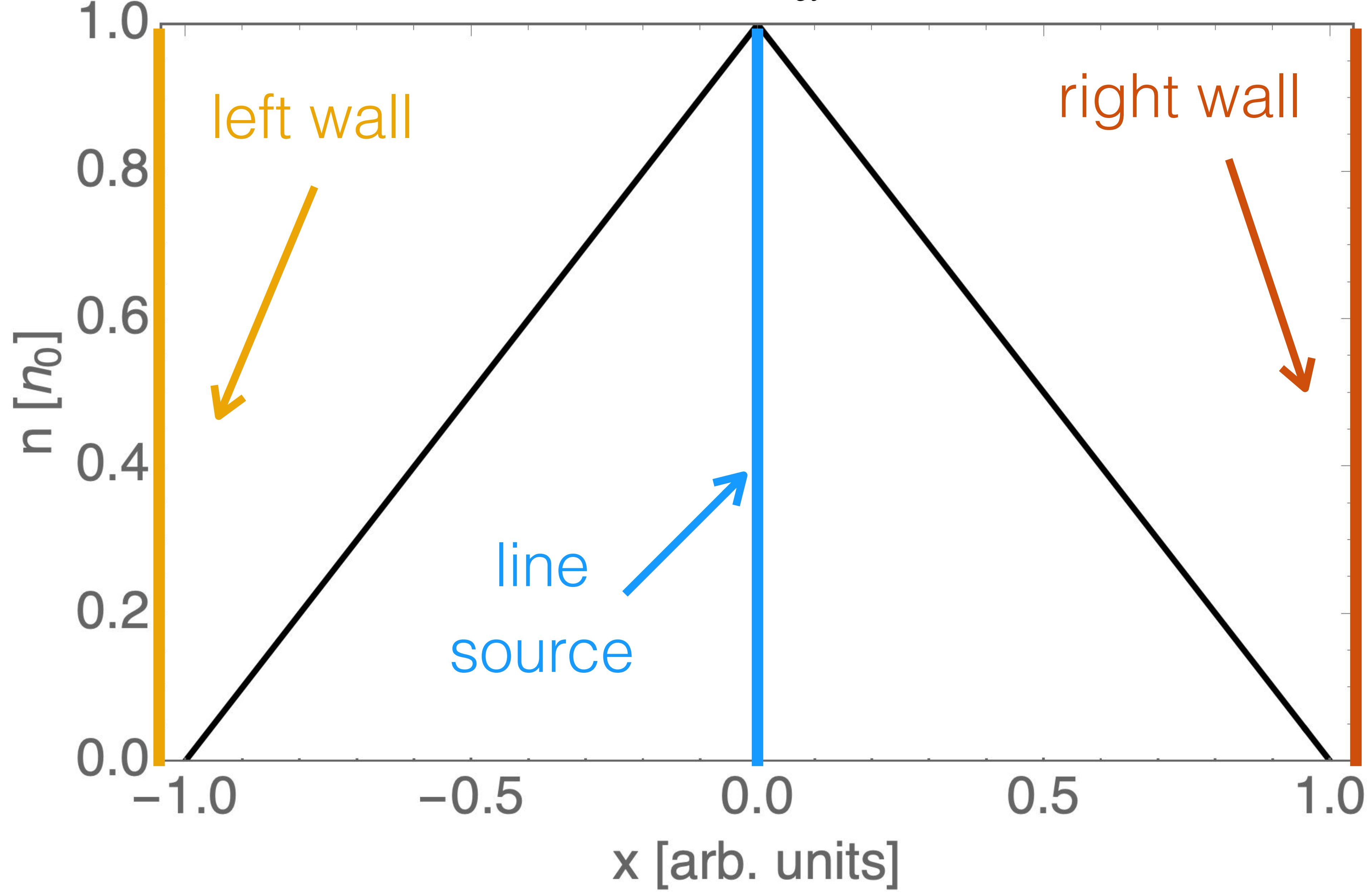
Example: plane source

$$\nabla^2 n = -\frac{Q}{D_a}\delta(0)$$



$$n = n_0\left(1 - \frac{|x|}{L}\right)$$

Chen, pp. 165-167

Recombination

Recombination processes

- Electron ion recombination can lower the plasma density

$$\frac{\partial n}{\partial t} = -|S(\mathbf{r})|$$

- Recombination between electron and ion requires at least one additional particle to conserve momentum

- Examples

  - Photon emission

$$M^+ + e \rightarrow M + h\nu$$

  - Neutral particle (3 body recombination)

$$M^+ + M + e \rightarrow M + M$$

  - Dissociation product - dissociative recombination

$$AB^+ + e \rightarrow A + B$$

Model

- Loss term proportional to $n_i n_e = n^2$
- Continuity equation reads (neglect diffusion):
- α is volume recombination coefficient $[m^3/s]$

$$\frac{\partial n}{\partial t} = -\alpha n^2$$

- Solution is:

$$\frac{1}{n(\mathbf{r},t)} = \frac{1}{n_0(\mathbf{r})} + \alpha t$$

- Distinction between decay and diffusion is possible because for large t $n \propto 1/(\alpha t)$!

Chen, pp. 167-169