Plasma Physics and Technology
Diffusion in weakly ionized plasmas with B

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Diffusion along and across B

Diffusion along B

Motions along B does not change motion.
Diffusive processes do not change!

$$\Gamma_z = \mu n E_z - D\frac{\partial n}{\partial z}$$

Particle flux along the direction of B.

Diffusion across B

No collisions

- no diffusion
- particles execute Larmor gyrations about lines of force.
- direction of single particle drifts can also be controlled to avoid collisions with walls of container.

Collisions

- particles move across lines of B.
- motion is still cyclotronic but with phase and **guiding center jumps**.
- guiding center performs random walk until reaching walls of container.
- random walk step is no longer λ but rather the Larmor radius r_L.
- diffusion (should) decrease with smaller r_L and thus with larger B

Dawson, pp 169-175

Diffusion across B: classical theoretical model

Fluid equation of motion for electrons and ions

Steady state

$$mn\cancel{\frac{\mathrm{d}\mathbf{v}_\perp}{\mathrm{d}t}} = \pm en\left(\mathbf{E} + \mathbf{v}_\perp \times \mathbf{B}\right) - KT\nabla n - mn\nu\mathbf{v}$$

Velocity components

$$v_x = \mu E_x - \frac{D}{n}\frac{\partial n}{\partial x} + \frac{\omega_c}{\nu}v_y$$

$$v_y = \mu E_y - \frac{D}{n}\frac{\partial n}{\partial y} - \frac{\omega_c}{\nu}v_x$$

Dawson, pp 169-175

Diffusion across B: classical theory

Fluid equation of motion for electrons and ions

Solve for v_x and v_y (where $\tau = 1/\nu$)

$$v_x\left(1+\omega_c^2\tau^2\right) = \mu E_x - \frac{D}{n}\frac{\partial n}{\partial x} + \omega_c^2\tau^2\frac{E_y}{B} \mp \omega_c^2\tau^2\frac{k_bT}{eB}\frac{1}{n}\frac{\partial n}{\partial y}$$

$$v_y\left(1+\omega_c^2\tau^2\right) = \mu E_y - \frac{D}{n}\frac{\partial n}{\partial y} - \omega_c^2\tau^2\frac{E_x}{B} \pm \omega_c^2\tau^2\frac{k_bT}{eB}\frac{1}{n}\frac{\partial n}{\partial x}$$

Last two terms: ExB and polarisation drifts

$$v_{E,x} = \frac{E_y}{B} \qquad v_{D,x} = \mp\frac{k_bT}{eB}\frac{1}{n}\frac{\partial n}{\partial y}$$

$$v_{E,y} = -\frac{E_x}{B} \qquad v_{D,y} = \pm\frac{k_bT}{eB}\frac{1}{n}\frac{\partial n}{\partial x}$$

Dawson, pp 169-175

Diffusion across B: classical theory

Define perpendicular mobility and diffusion coefficients

$$\mu_\perp = \frac{\mu}{1 + \omega_c^2 \tau^2}$$

$$D_\perp = \frac{D}{1 + \omega_c^2 \tau^2}$$

Drift velocity

$$\mathbf{v}_\perp = \mu_\perp \mathbf{E} - D_\perp \frac{\nabla n}{n} + \frac{\mathbf{v}_E + \mathbf{v}_D}{1 + \omega_c^2 / \nu^2}$$

Physical significance

Transverse velocity

- Drifts perpendicular to field and density gradients
  - Reduced by collisions with neutrals.
- Drifts parallel to field and density gradients
  - Same form of un-magnetised scenario
  - Reduced by $1 + \omega_c^2 \tau^2$

Dawson, pp 169-175

Diffusion across B: classical theory

Alternative expressions

$$\omega_c \tau = \omega_c / \nu = \mu B \simeq \frac{\lambda}{r_L}$$

Limit of $\omega_c^2 \tau^2 \gg 1$

$$D_\perp = \frac{k_b T}{m\nu} \frac{1}{\omega_c^2 \tau^2} = \frac{k_b T \nu}{m \omega_c^2}$$

Significance and implications

Diffusion parallel to B

- Collisions slow down diffusion
  $D \propto 1/\nu$
- Heavier species diffuse slowly
  $D \propto m^{-1/2}$

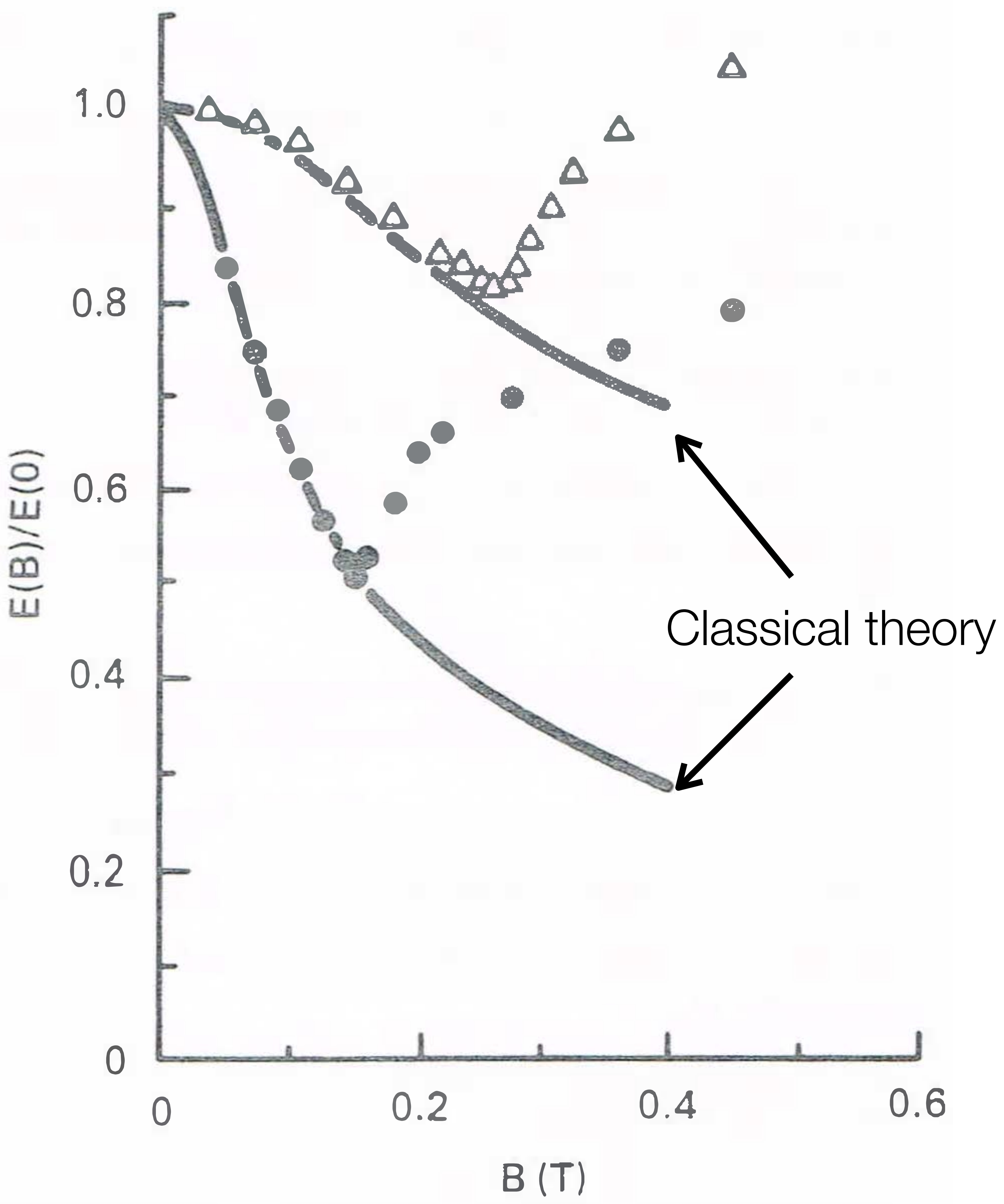
Diffusion perpendicular to B

- Collisions require for particles to diffuse across B
  $D_\perp \propto \nu$
- Lighter species diffuse slower because their Larger gyrations are smaller than for lighter particles
  $D_\perp \propto m^{1/2}$

Dawson, pp 169-175

Diffusion across B: classical theory vs experiments

Experimental results



Normalised longitudinal E field measured as a function of B at two different pressures [F.C. Hoh and B. Lehnert, Phys. Fluids 3 600 (1960)].

Plasma confinement

Implications for plasma confinement

- Diffusion in agreement with theory but only when B is small enough.
- After that B enhances diffusion!
- Instabilities not considered in theory arise for large values of B

Dawson, pp 169-175