Plasma Physics and Technology
Electromagnetic waves in magnetised plasmas

TÉCNICO LISBOA

Mestrado Integrado em Engenharia Física Tecnológica

Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico

How to study plasmas

- single particle motion
  - simple but powerful analysis
  - enables to investigate key waves and instabilities in plasma physics

- plasma kinetic equations
  - general approach
  - can be solved using computer programs

- **fluid equations**
  - plasma waves and instabilities
  - interaction with electromagnetic waves

Electromagnetic waves in magnetised plasmas

Relevant equations

- Electron force equation

$$m_e n_0 \frac{\partial \mathbf{v}_e}{\partial t} = -en_0\mathbf{E}_1 - en_0\mathbf{v}_e \times \mathbf{B}_0$$

- Faraday's law

$$\nabla \times \mathbf{E}_1 = -\frac{\partial \mathbf{B}}{\partial t}$$

- Ampere's law

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}_e + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

- Electronic current

$$\mathbf{j}_e = -en_0\mathbf{v}_e$$

Nicholson, pp. 151

Ordinary (and extraordinary waves)

Relevant equations

- Electron force equation

$$m_e n_0 \frac{\partial \mathbf{v}_e}{\partial t} = -e n_0 \mathbf{E}_1 - e n_0 \mathbf{v}_e \times \mathbf{B}_0$$

- Faraday's law

$$\nabla \times \mathbf{E}_1 = -\frac{\partial \mathbf{B}}{\partial t}$$

- Ampere's law

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}_e + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

- Electronic current

$$\mathbf{j}_e = -e n_0 \mathbf{v}_e$$

Underlying assumptions/limits

- **Key assumption** $\mathbf{k} \perp \mathbf{B}_0$
- Two possibilities:
  - $\mathbf{E} \| \mathbf{B}_0$ ordinary wave
  - $\mathbf{E} \perp \mathbf{B}_0$ extraordinary wave
- Ordinary wave

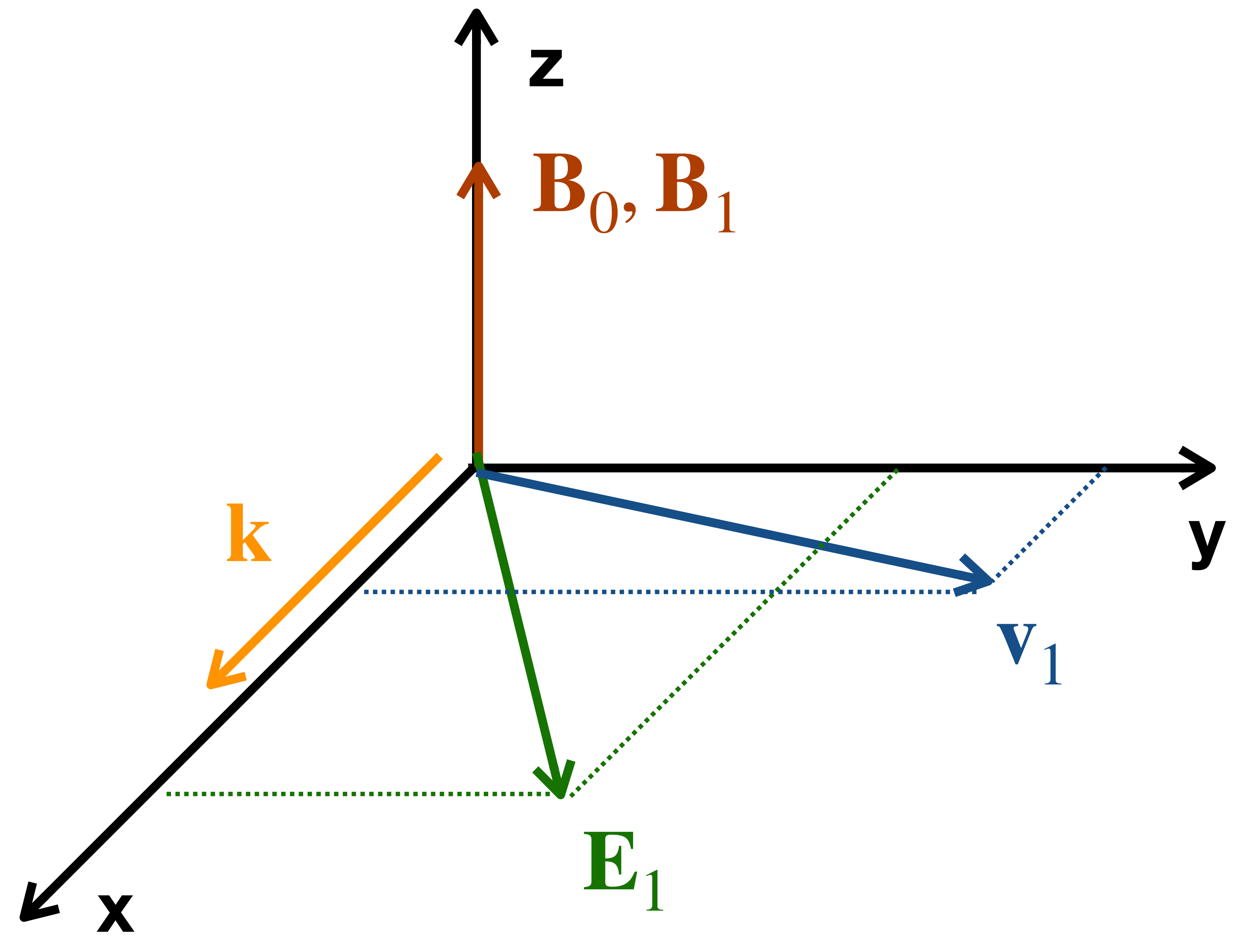
$$\mathbf{v}_e \times \mathbf{B}_0 = 0 \Rightarrow \omega^2 = \omega_p^2 + k^2 c^2$$

Same as in un-magnetised plasma!

Nicholson, pp. 151

Extraordinary waves - refractive index

Geometry

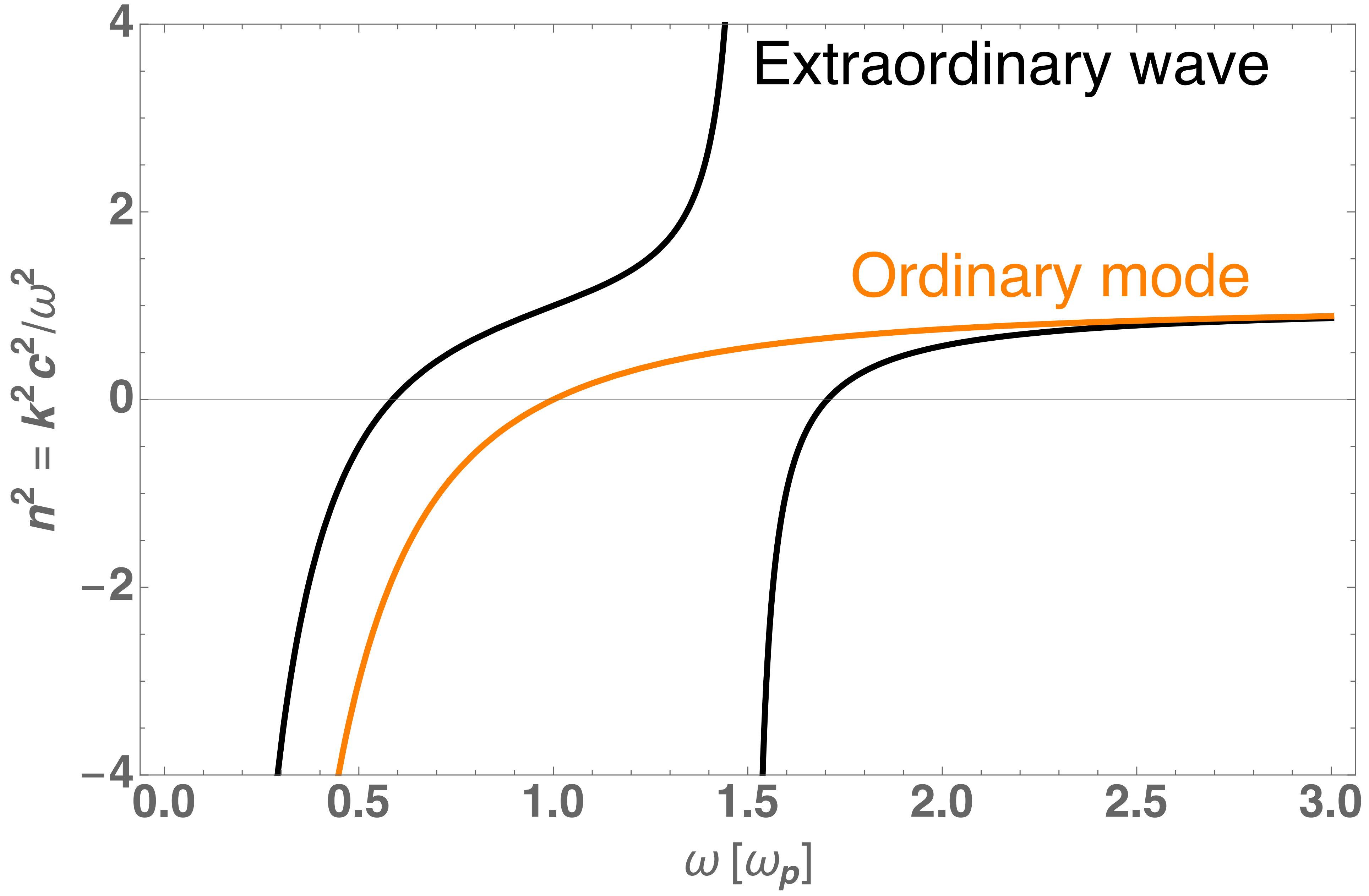


- $\mathbf{E}_1 = E_x\mathbf{e}_x + E_y\mathbf{e}_y$
- $\mathbf{v}_1 = v_x\mathbf{e}_x + v_y\mathbf{e}_y$
- $\mathbf{B}_0 = B_0\mathbf{e}_z$
- $\mathbf{B}_1 = B_1\mathbf{e}_z$

Index of refraction

- After a few calculations…

$$n^2 \equiv \frac{k^2c^2}{\omega^2} = 1 + \frac{\omega_p^2}{\omega^2}\frac{\omega_p^2 - \omega^2}{\omega^2 - \omega_{UH}^2}$$



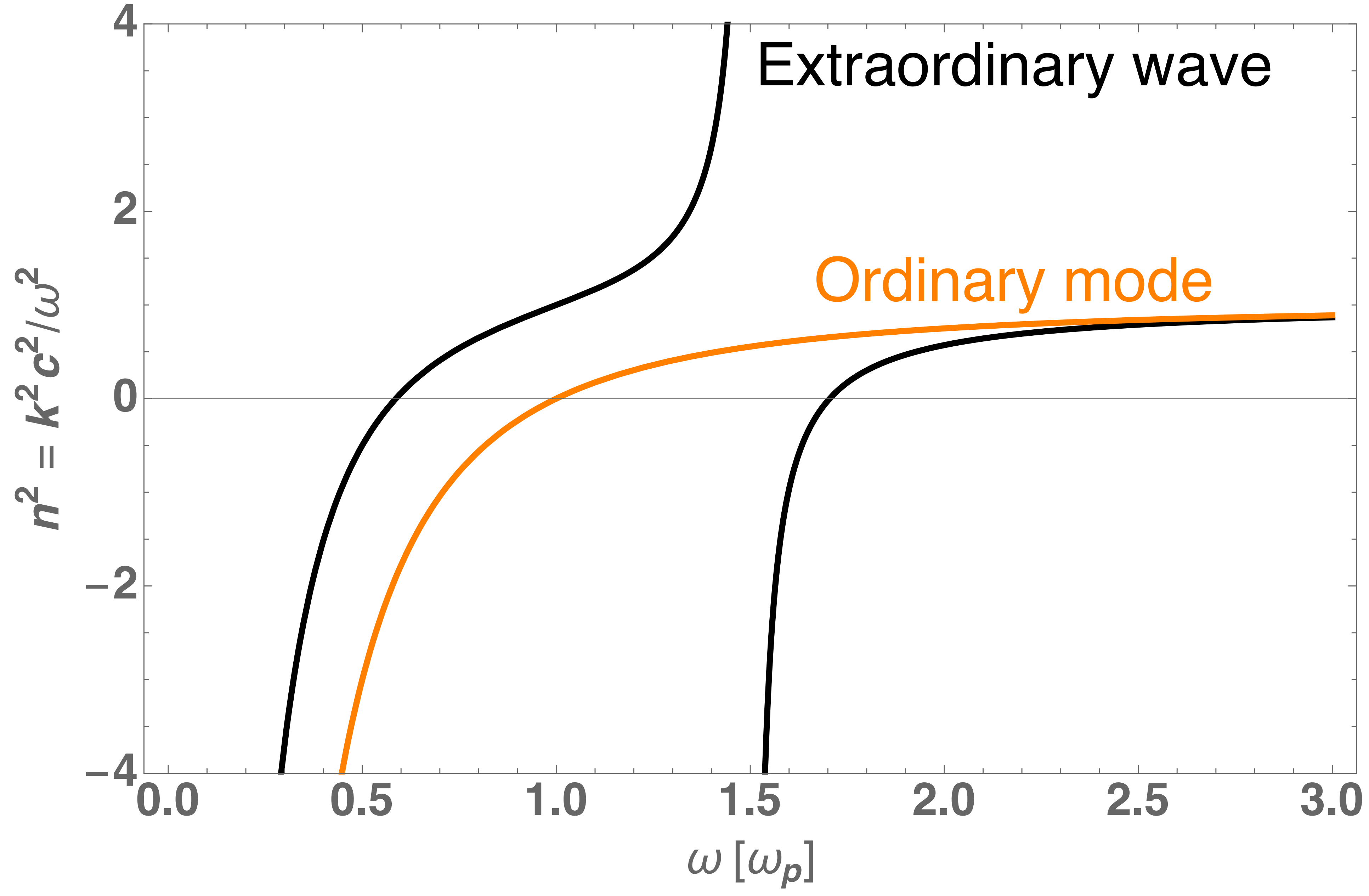
Nicholson, pp. 152-156

Extraordinary waves - physical picture

Index of refraction

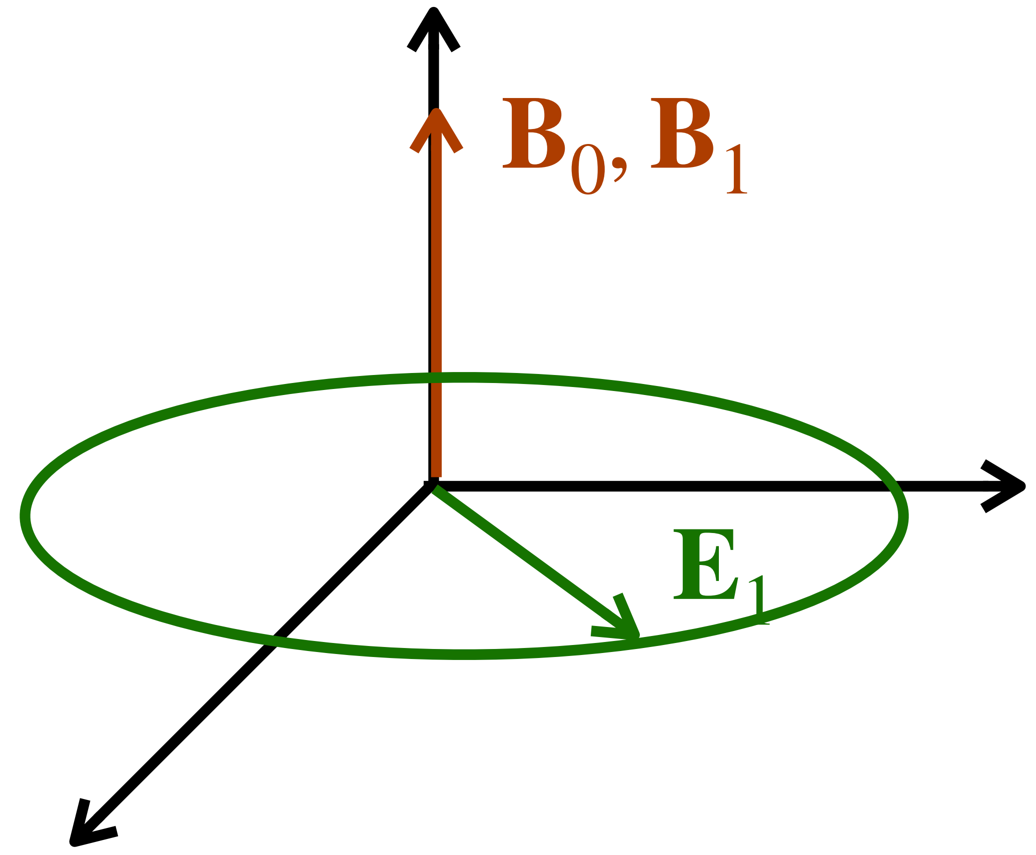
- After a few calculations…

$$n^2 \equiv \frac{k^2c^2}{\omega^2} = 1 + \frac{\omega_p^2}{\omega^2}\frac{\omega_p^2 - \omega^2}{\omega^2 - \omega_{UH}^2}$$



Physical picture

- partially longitudinal, partially transverse wave
- the components of the electric field and are not in phase. Polarisation rotates in the direction of right hand rule!



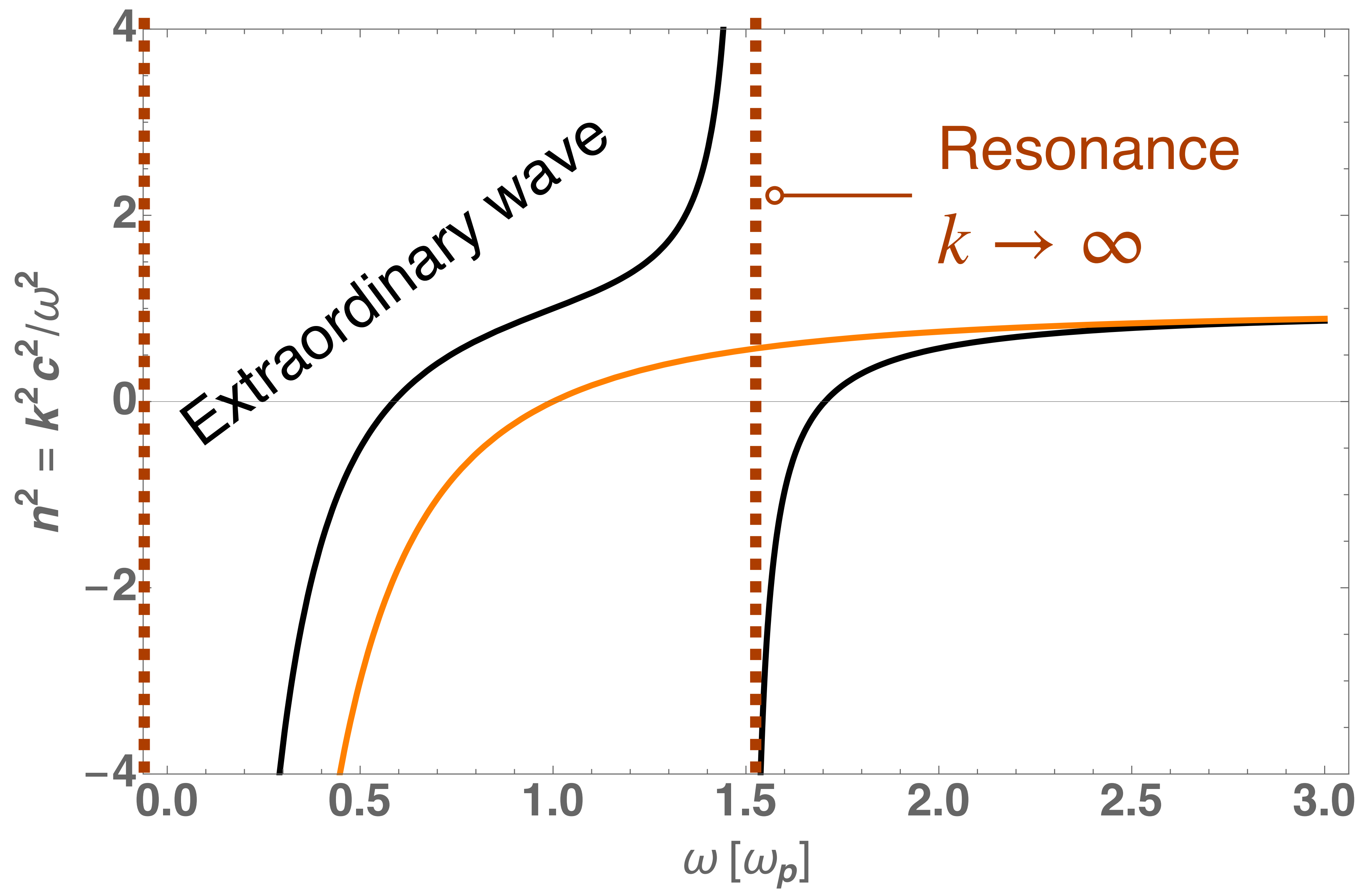
Nicholson, pp. 152-156

Extraordinary waves - Resonances

Index of refraction

- After a few calculations…

$$n^2 \equiv \frac{k^2c^2}{\omega^2} = 1 + \frac{\omega_p^2}{\omega^2}\frac{\omega_p^2 - \omega^2}{\omega^2 - \omega_{UH}^2}$$



Resonance: $k \to \infty$

- After a few short calculations…

$$\omega_{L/R} = 0$$

$$\omega = \omega_{UH}$$

- Latter scenario

Electromagnetic wave continuously accelerates electrons eventually heating up the plasma!

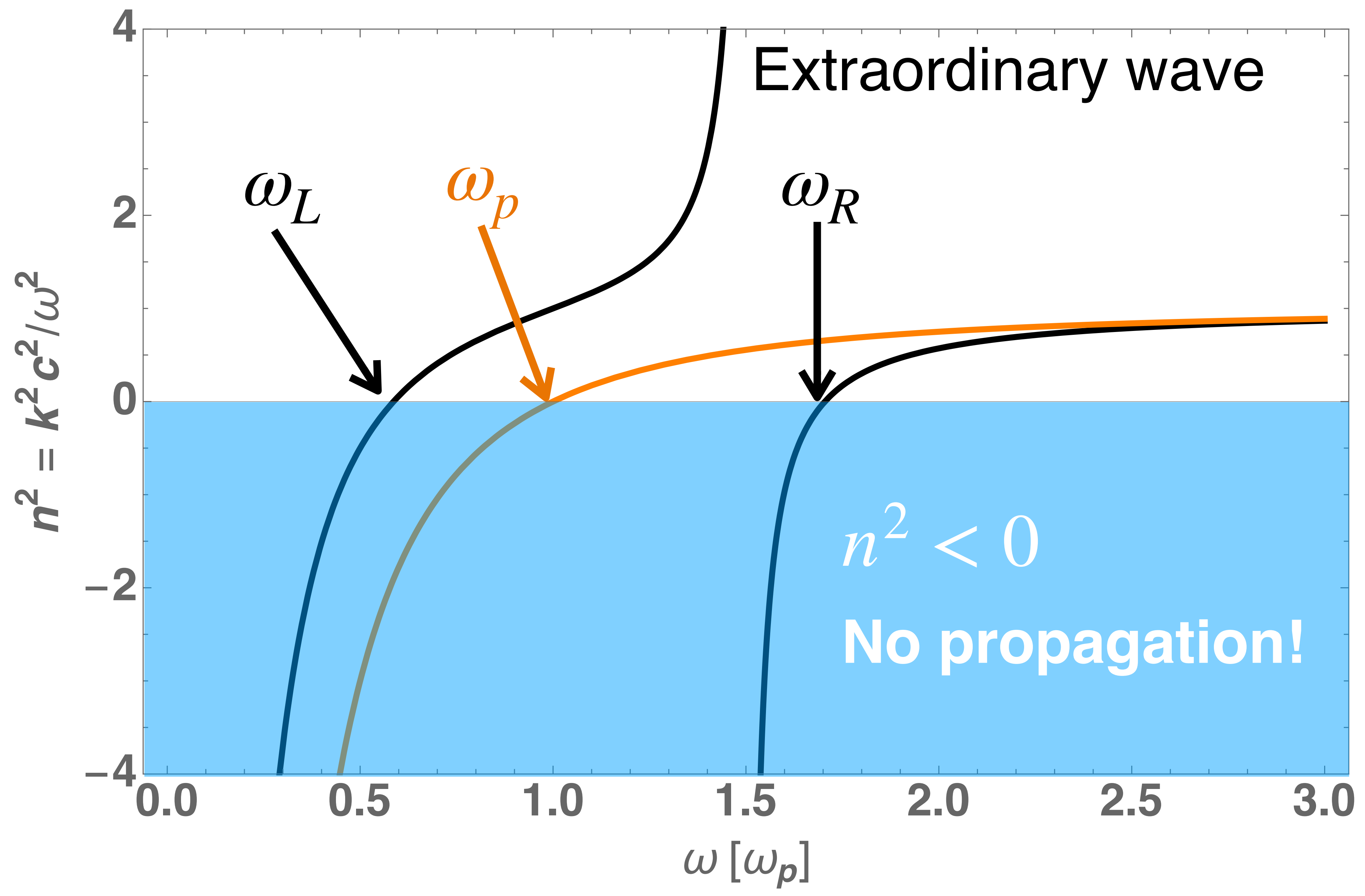
Nicholson, pp. 152-156

Extraordinary waves - Cut-offs

Index of refraction

- After a few calculations…

$$n^2 \equiv \frac{k^2c^2}{\omega^2} = 1 + \frac{\omega_p^2}{\omega^2}\frac{\omega_p^2 - \omega^2}{\omega^2 - \omega_{UH}^2}$$



Cut-offs: $k \rightarrow 0$

- Two cutoff frequencies

$$\omega_{L/R} = \pm\frac{\Omega_e}{2} + \sqrt{\omega_p^2 + \frac{\Omega_e^2}{4}}$$

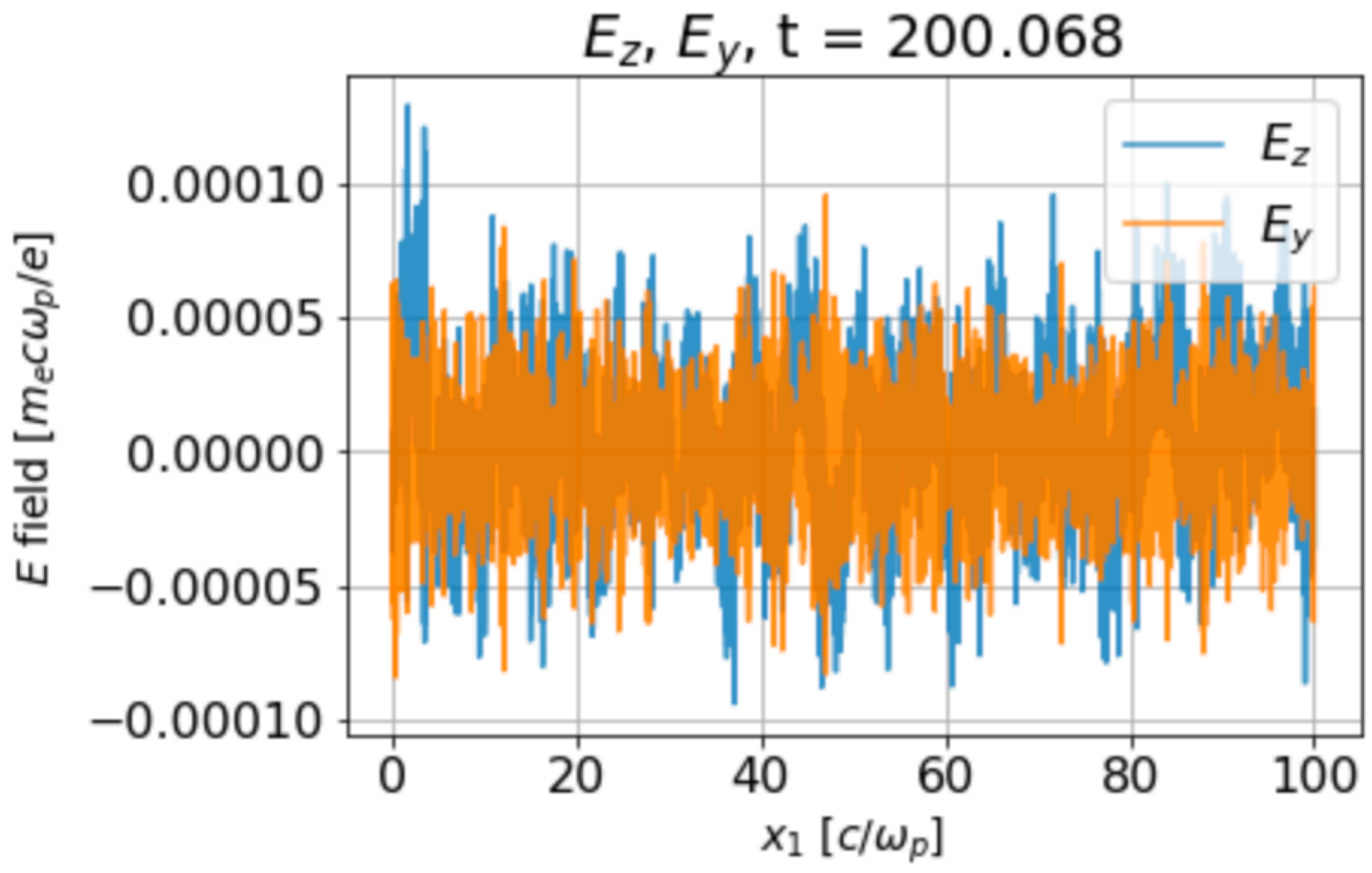
- $\omega_{R/L}$ are the two cut-off frequencies (meaning of designation will become clearer soon)

Nicholson, pp. 152-156

Ordinary and extraordinary modes - numerical simulation

Simulation parameters

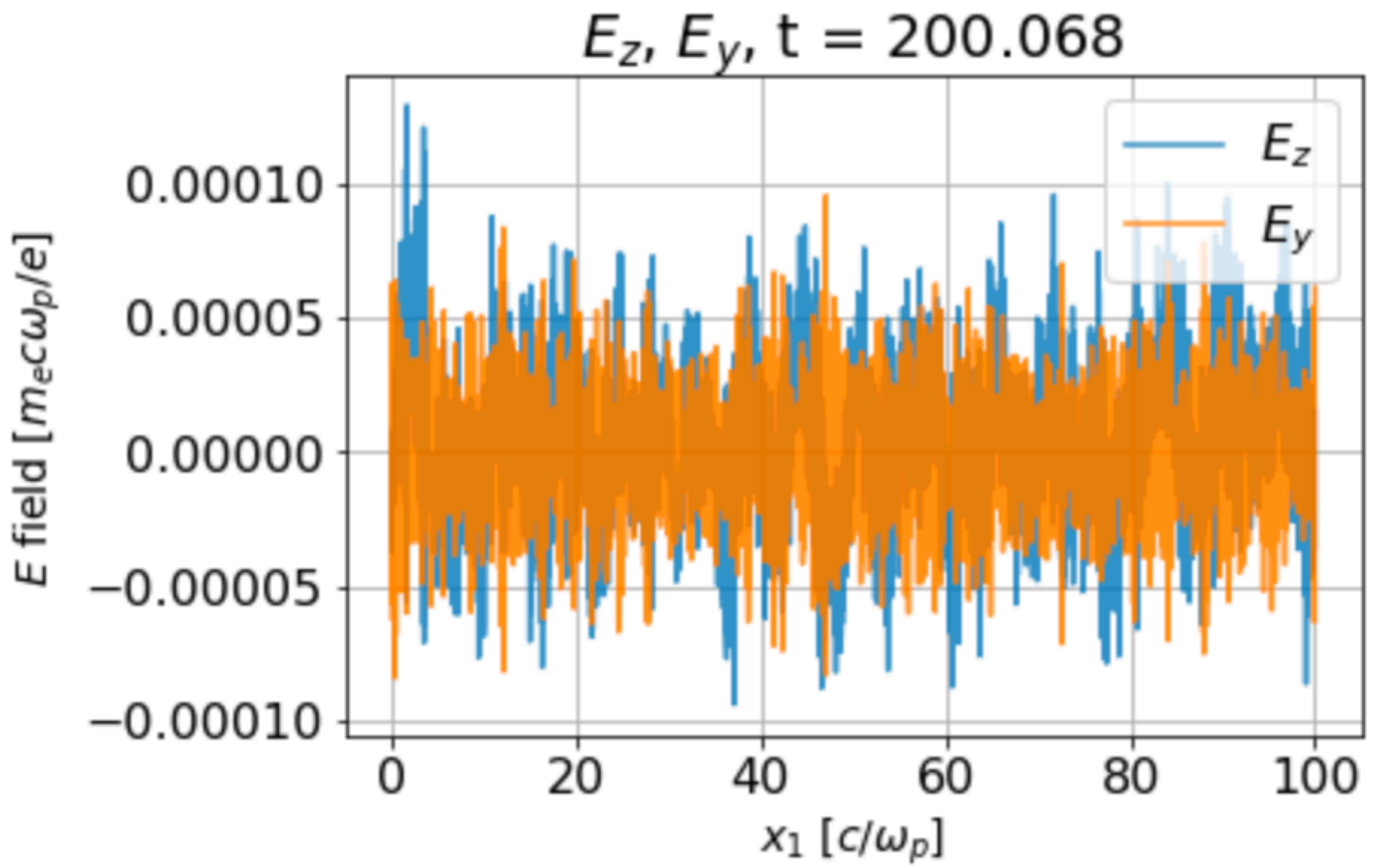
- Magnetised plasma with $\omega_c = 0.5\omega_p$
- External magnetic field along $\mathbf{e}_z$
- Simulation along $\mathbf{e}_x$



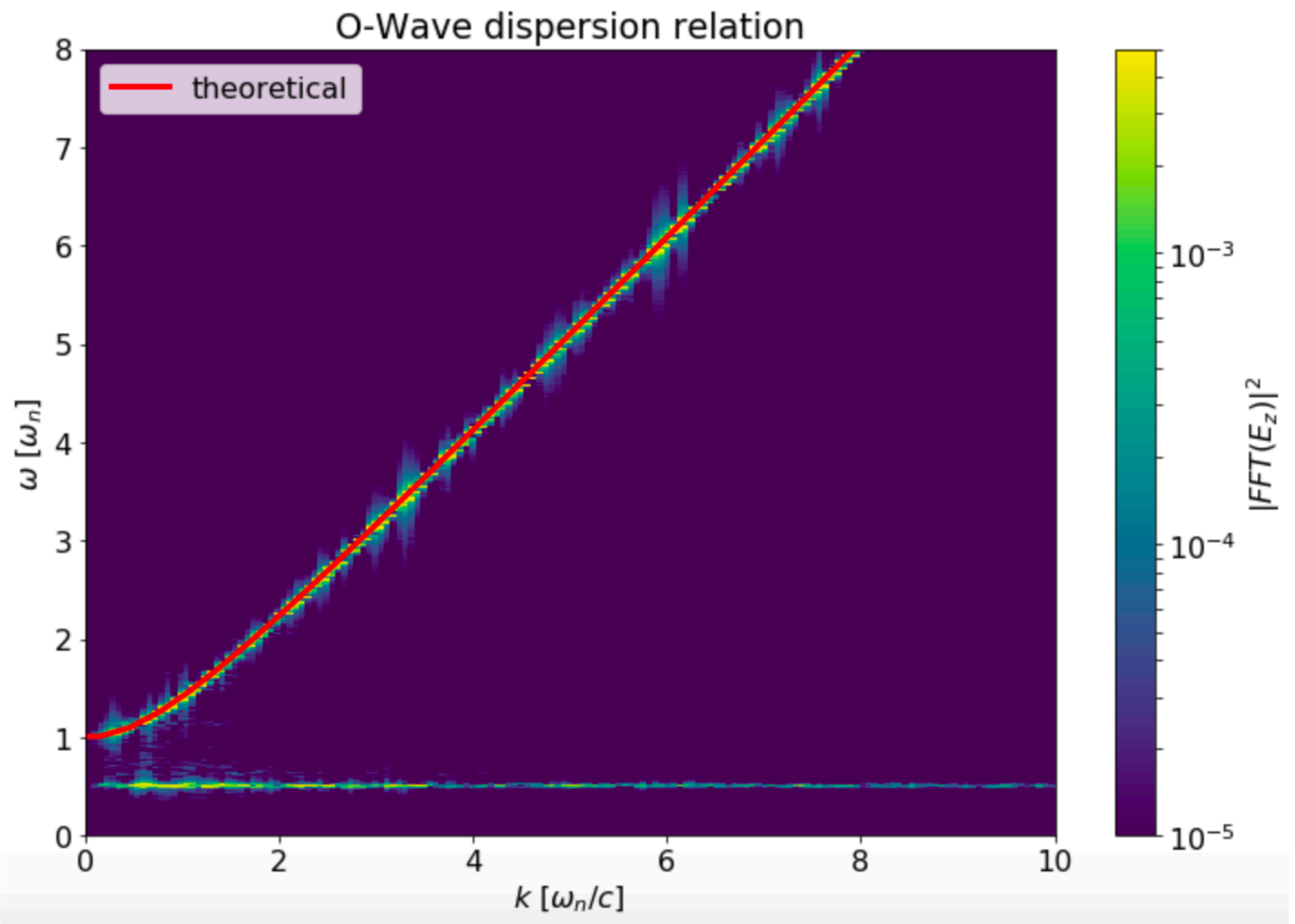
Ordinary modes - numerical simulation

Simulation parameters

- Magnetised plasma with $\omega_c = 0.5\omega_p$
- External magnetic field along $\mathbf{e}_z$
- Simulation along $\mathbf{e}_x$



Ordinary mode

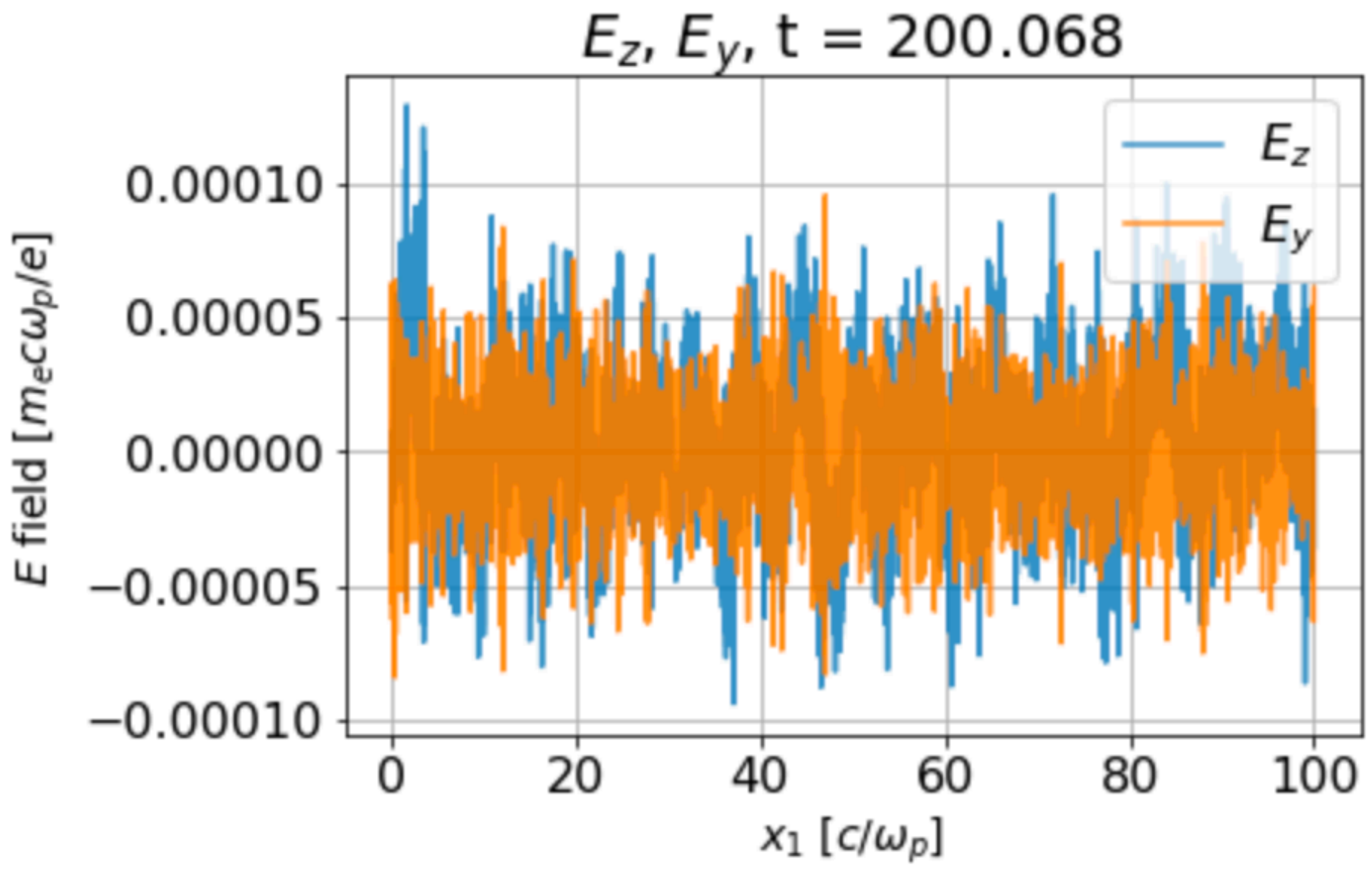


What is the polarisation direction of the ordinary mode?

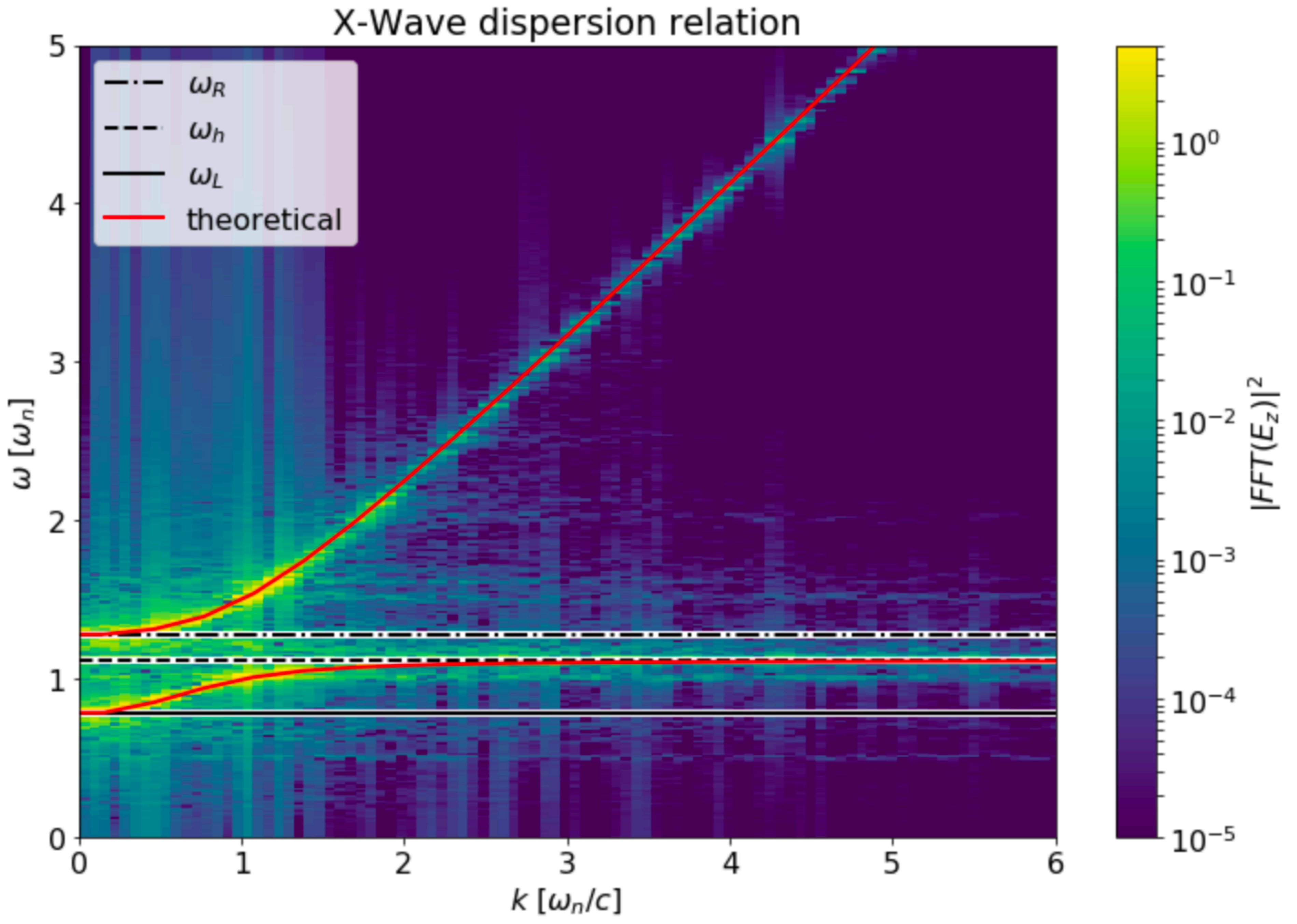
Extraordinary modes - numerical simulation

Simulation parameters

- Magnetised plasma with $\omega_c = 0.5\omega_p$
- External magnetic field along $\mathbf{e}_z$
- Simulation along $\mathbf{e}_x$



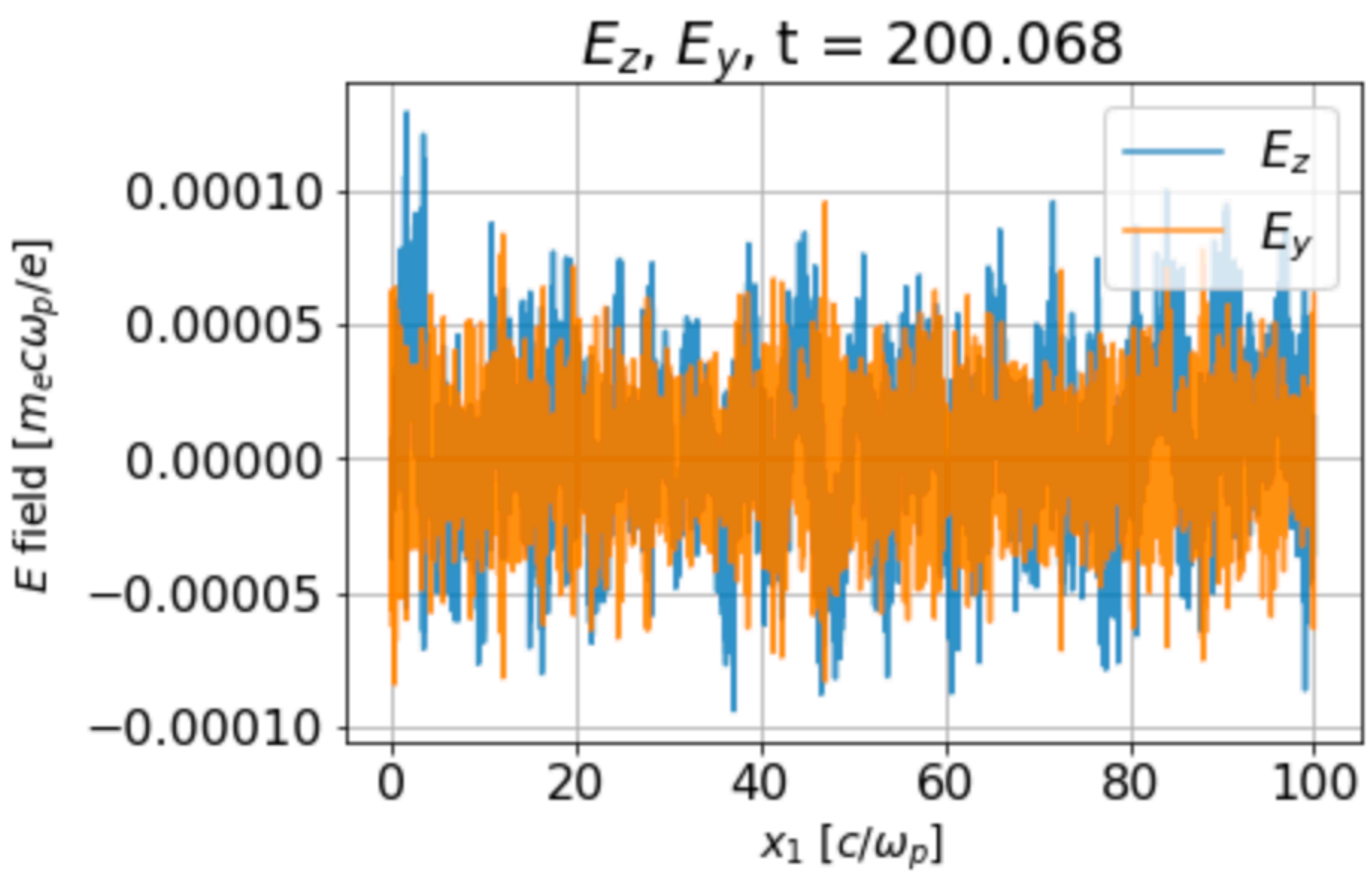
Extra-ordinary mode (E_y)



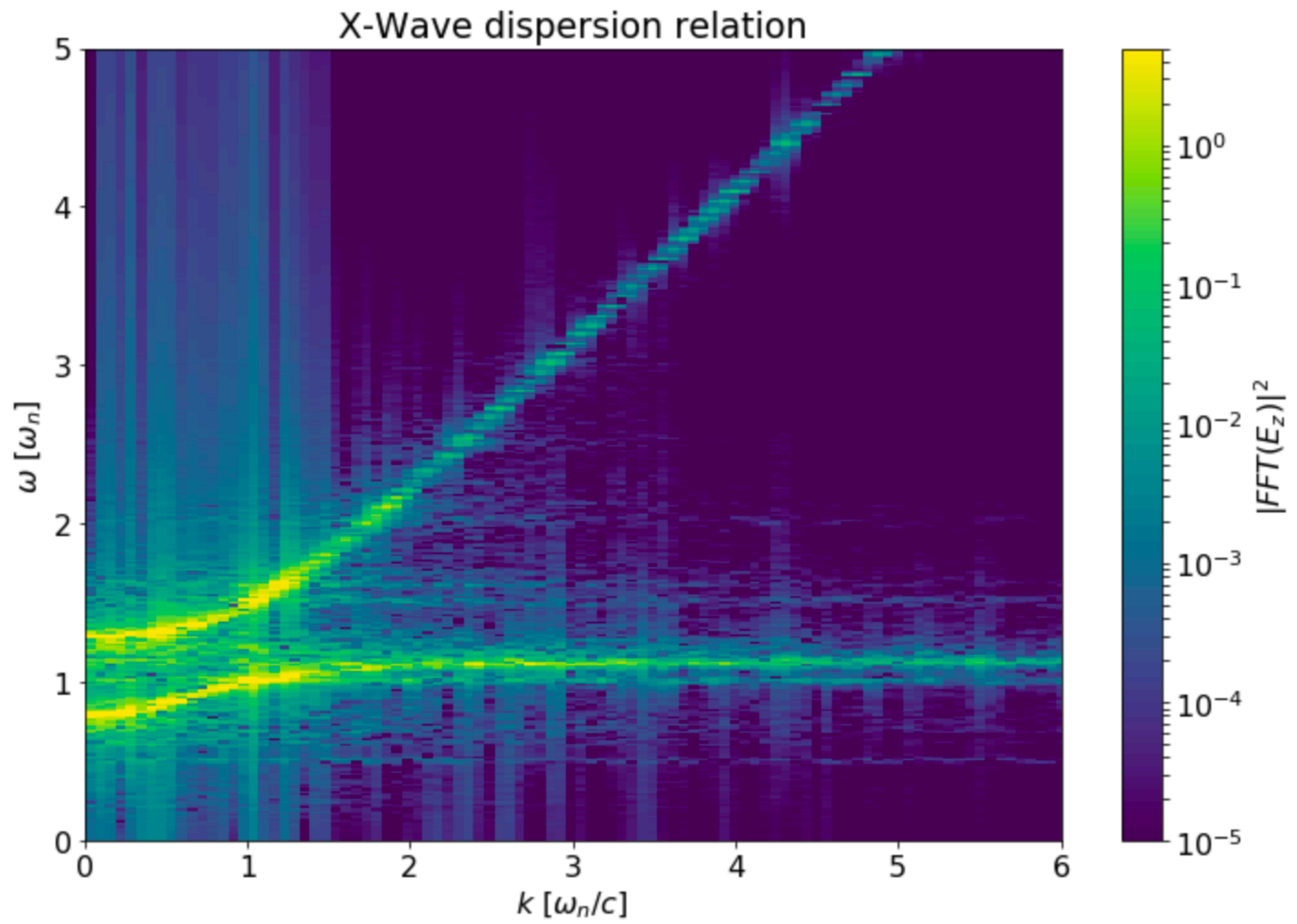
Extraordinary waves - puzzling question??

Simulation parameters

- Magnetised plasma with $\omega_c = 0.5\omega_p$
- External magnetic field along $\mathbf{e}_z$
- Simulation along $\mathbf{e}_x$



Extra-ordinary mode $(\mathbf{E_y})$

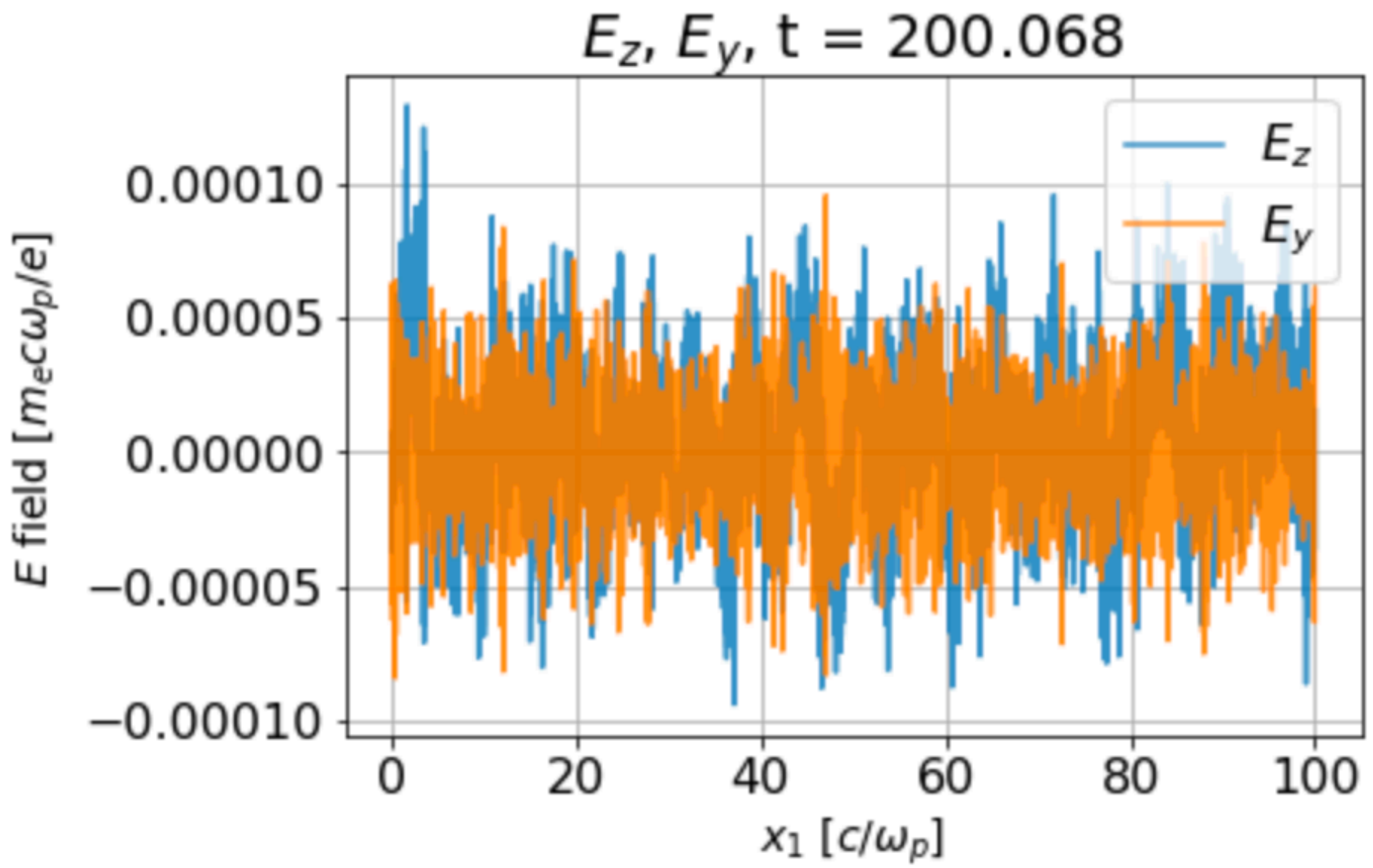


Why does the signal weakens when $\omega \to \omega_h$

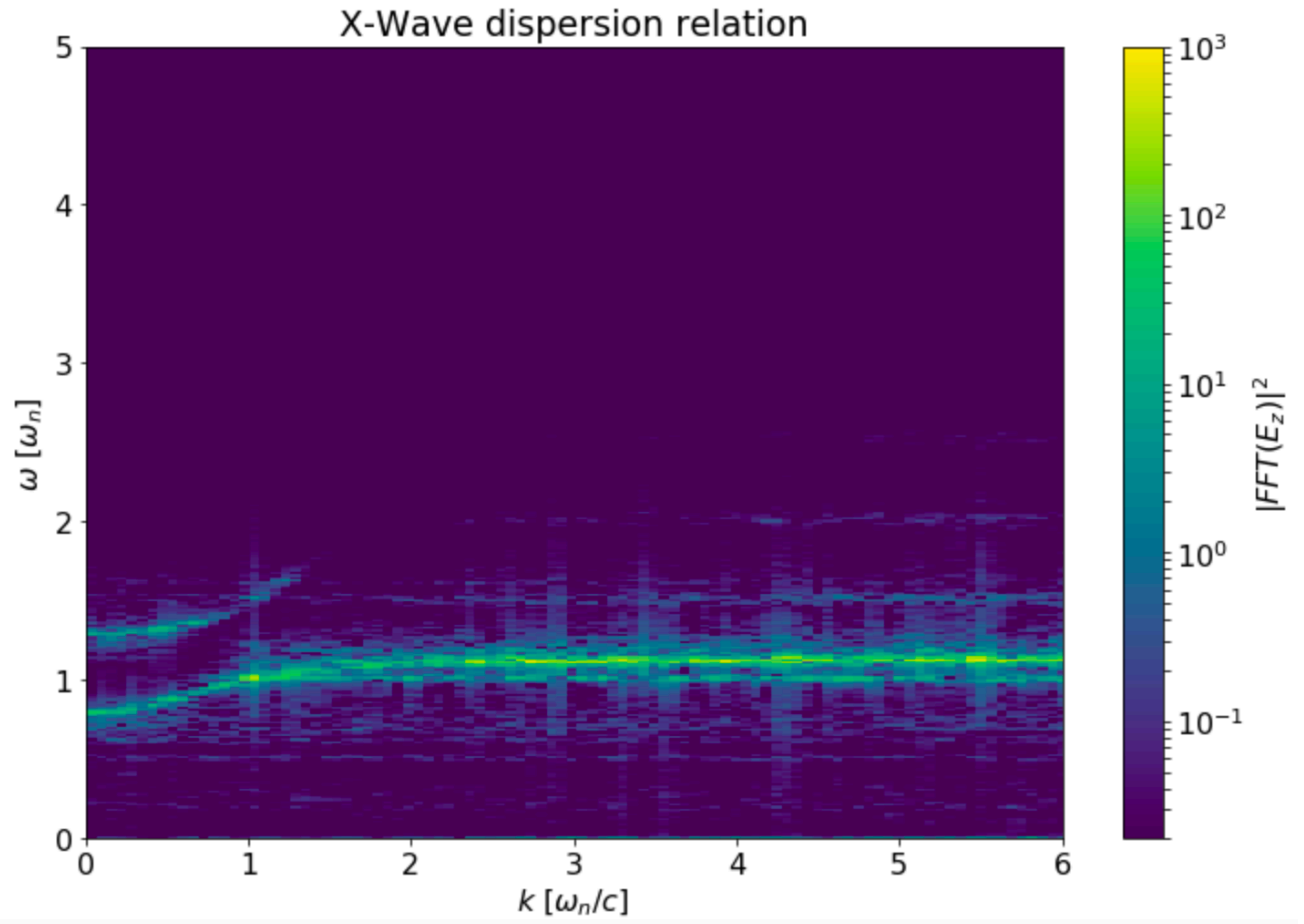
Extraordinary waves - puzzling question!

Simulation parameters

- Magnetised plasma with $\omega_c = 0.5\omega_p$
- External magnetic field along $\mathbf{e}_z$
- Simulation along $\mathbf{e}_x$



Extra-ordinary mode ($\mathbf{E_x}$)



What is the signal weakens when $\omega > \omega_R$

Electromagnetic wave propagation along B

Relevant equations

- Electron force equation

$$m_e n_0 \frac{\partial \mathbf{v}_e}{\partial t} = -e n_0 \mathbf{E}_1 - e n_0 \mathbf{v}_e \times \mathbf{B}_0$$

- Faraday's law

$$\nabla \times \mathbf{E}_1 = -\frac{\partial \mathbf{B}}{\partial t}$$

- Ampere's law

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}_e + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

- Electronic current

$$\mathbf{j}_e = -e n_0 \mathbf{v}_e$$

Underlying assumptions/limits

- **Key assumption $\mathbf{k} || \mathbf{B}_0$**

- Two possibilities:
  - $\mathbf{E} || \mathbf{B}_0$ ordinary wave
  - $\mathbf{E} \perp \mathbf{B}_0$ extraordinary wave

- Ordinary wave

$$\mathbf{v}_e \times \mathbf{B}_0 = 0 \Rightarrow \omega^2 = \omega_p^2 + k^2 c^2$$

Same as in un-magnetised plasma!

Nicholson, pp. 156-161

Electromagnetic wave propagation along B

Relevant equations

- Electron force equation

$$m_e n_0 \frac{\partial \mathbf{v}_e}{\partial t} = - e n_0 \mathbf{E}_1 - e n_0 \mathbf{v}_e \times \mathbf{B}_0$$

- Faraday's law

$$\nabla \times \mathbf{E}_1 = - \frac{\partial \mathbf{B}}{\partial t}$$

- Ampere's law

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}_e + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

- Electronic current

$$\mathbf{j}_e = - e n_0 \mathbf{v}_e$$

Underlying assumptions/limits

- **Key assumption $\mathbf{k} || \mathbf{B}_0$**

- Two possibilities:
  - $\mathbf{E} || \mathbf{B}_0$ ordinary wave
  - $\mathbf{E} \perp \mathbf{B}_0$ extraordinary wave

- Ordinary wave

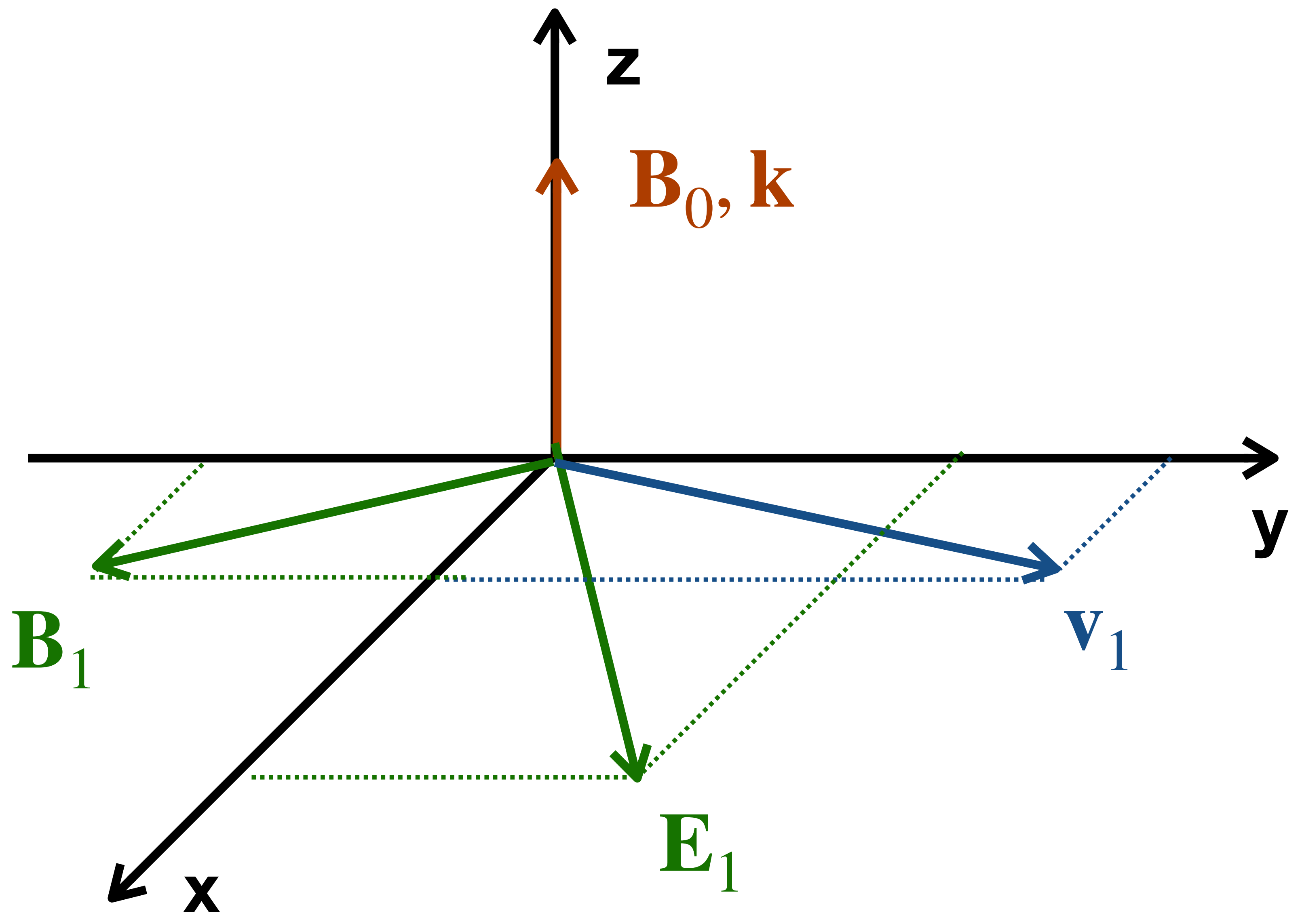
$$\mathbf{v}_e \times \mathbf{B}_0 = 0 \Rightarrow \omega^2 = \omega_p^2 + k^2 c^2$$

Same as in un-magnetised plasma!

Nicholson, pp. 156-161

Propagation along B - refractive index

Geometry

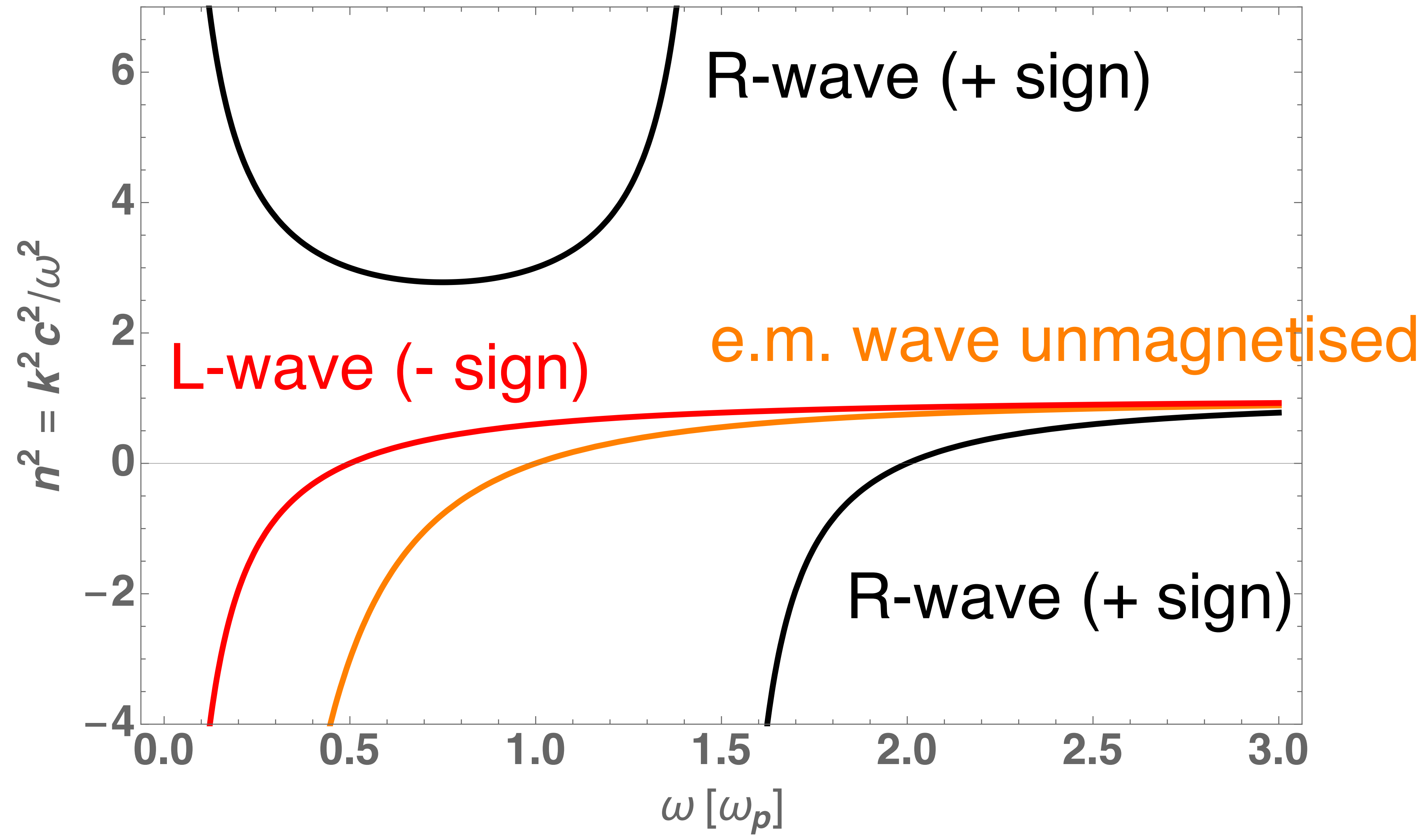


- $\mathbf{E}_1 = E_x\mathbf{e}_x + E_y\mathbf{e}_y$
- $\mathbf{v}_1 = v_x\mathbf{e}_x + v_y\mathbf{e}_y$
- $\mathbf{B}_0 = B_0\mathbf{e}_z$
- $\mathbf{B}_1 = B_x\mathbf{e}_x + B_y\mathbf{e}_y$
- $\mathbf{k}\|\mathbf{e}_z$

Index of refraction

- After a few calculations…

$$n^2 \equiv \frac{k^2c^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\frac{1}{1 \pm \frac{\Omega_e}{\omega}}$$



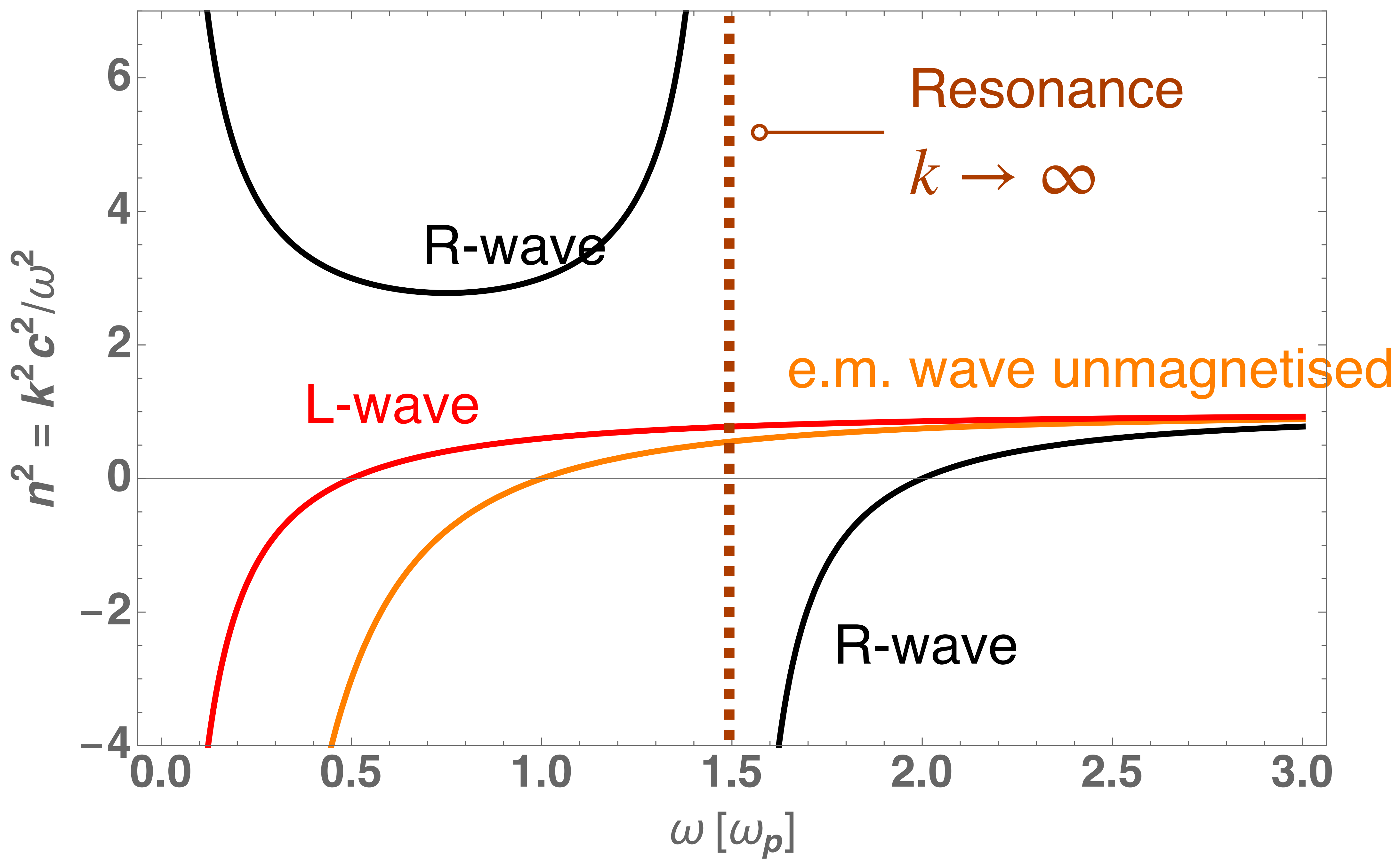
Nicholson, pp. 156-161

Propagation along B - refractive index

Index of refraction

- After a few calculations…

$$n^2 \equiv \frac{k^2c^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\frac{1}{1 \pm \frac{\Omega_e}{\omega}}$$



Resonance: $k \to \infty$

- R-wave resonance (wave polarisation rotates in the direction of electron cyclotron motion):
  - $\omega = |\Omega_e|$
- L-wave resonance:
  - None

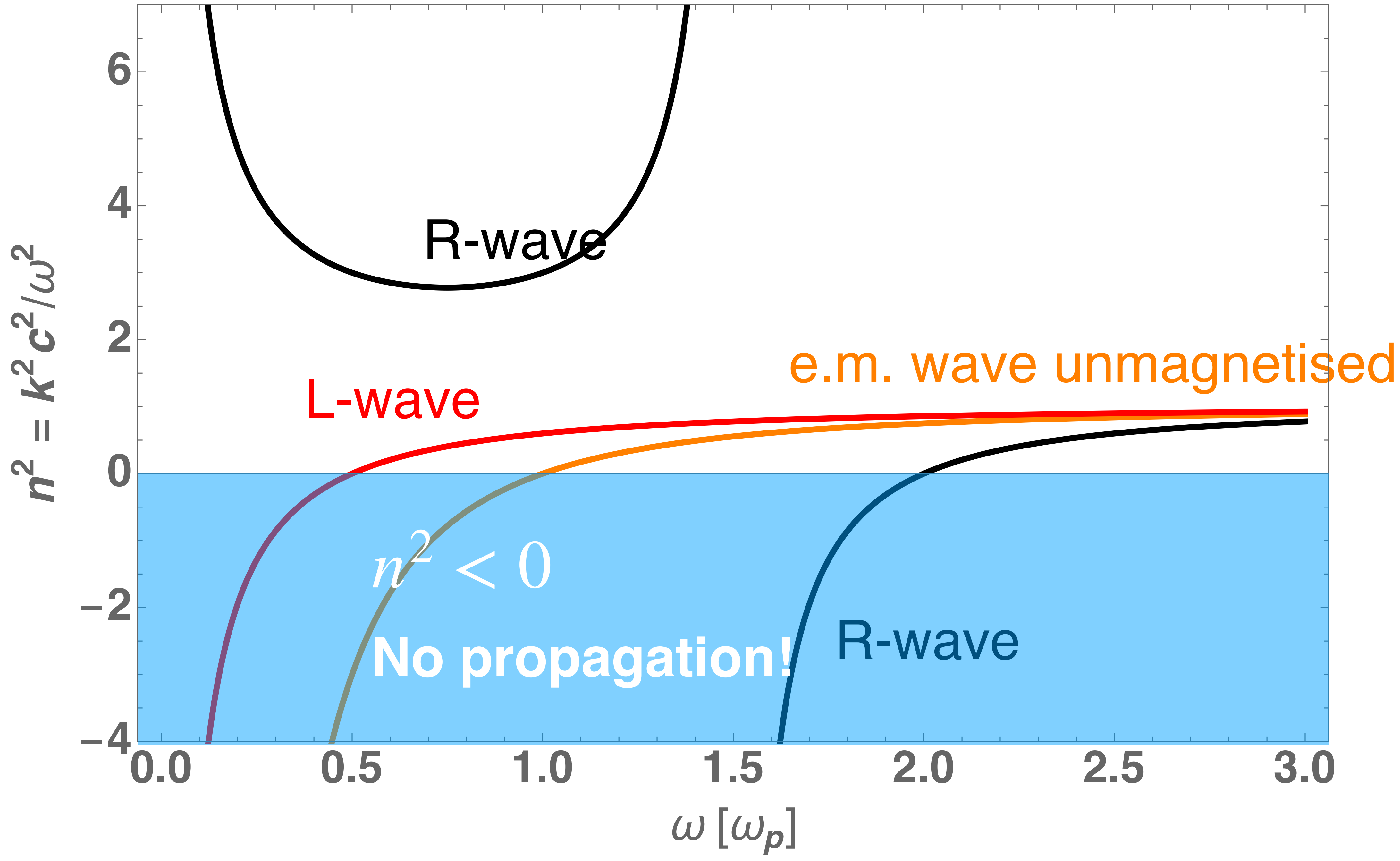
Nicholson, pp. 156-161

Propagation along B - refractive index

Index of refraction

- After a few calculations…

$$n^2 \equiv \frac{k^2c^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\frac{1}{1 \pm \frac{\Omega_e}{\omega}}$$



Cutoff: $k \to 0$

$$\omega_{R/L} = \mp\frac{\Omega_e}{2} + \left(\frac{\Omega_e^2}{4} + \omega_p^2\right)^{1/2}$$

These correspond exactly to the cutoffs for the extraordinary mode and hence the names $\omega_{R/L}$

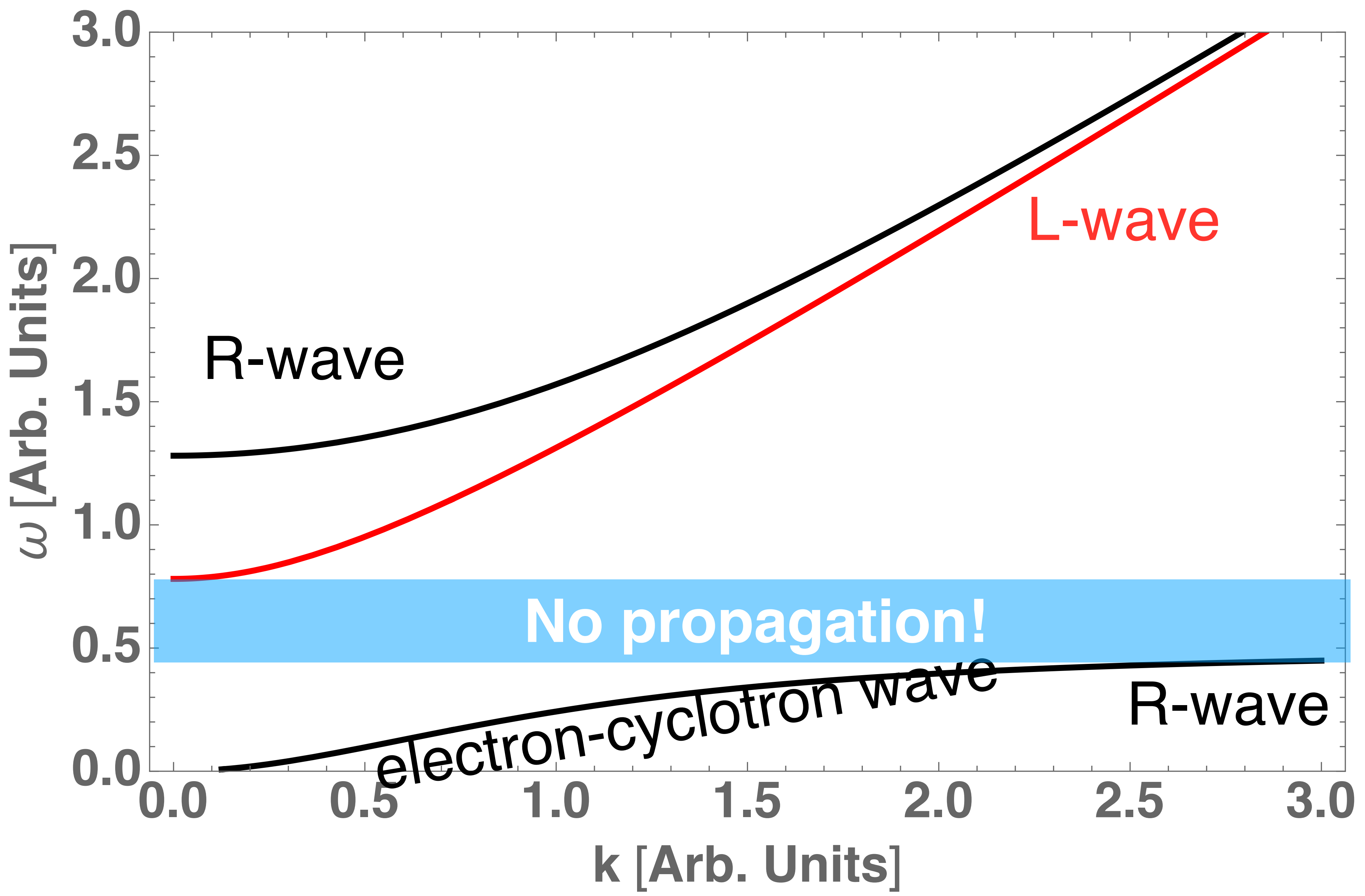
Ordering of resonant and cut-off frequencies

- Right waves: $\omega_r > |\Omega_e|$ (recall $\Omega_e < 0$!)
- Left waves: $\omega_L > |\Omega_e|$ or $\omega_L < |\Omega_e|$ (depending on how much is the plasma frequency.

Nicholson, pp. 156-161

Propagation along B - whistler mode

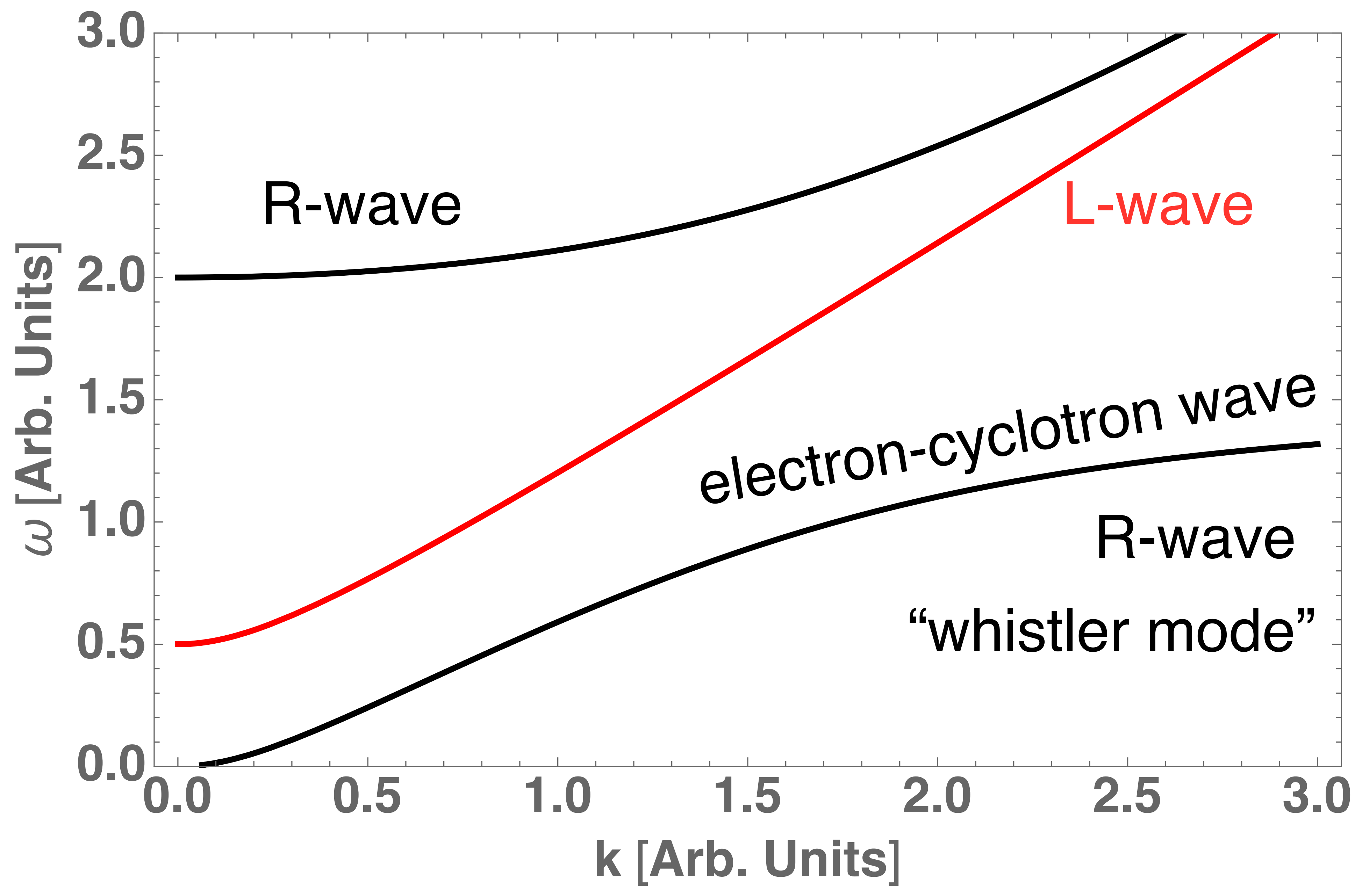
$\omega_L > |\Omega_e|$



Nicholson, pp. 156-161

Propagation along B - whistler mode

$\omega_L < |\Omega_e|$



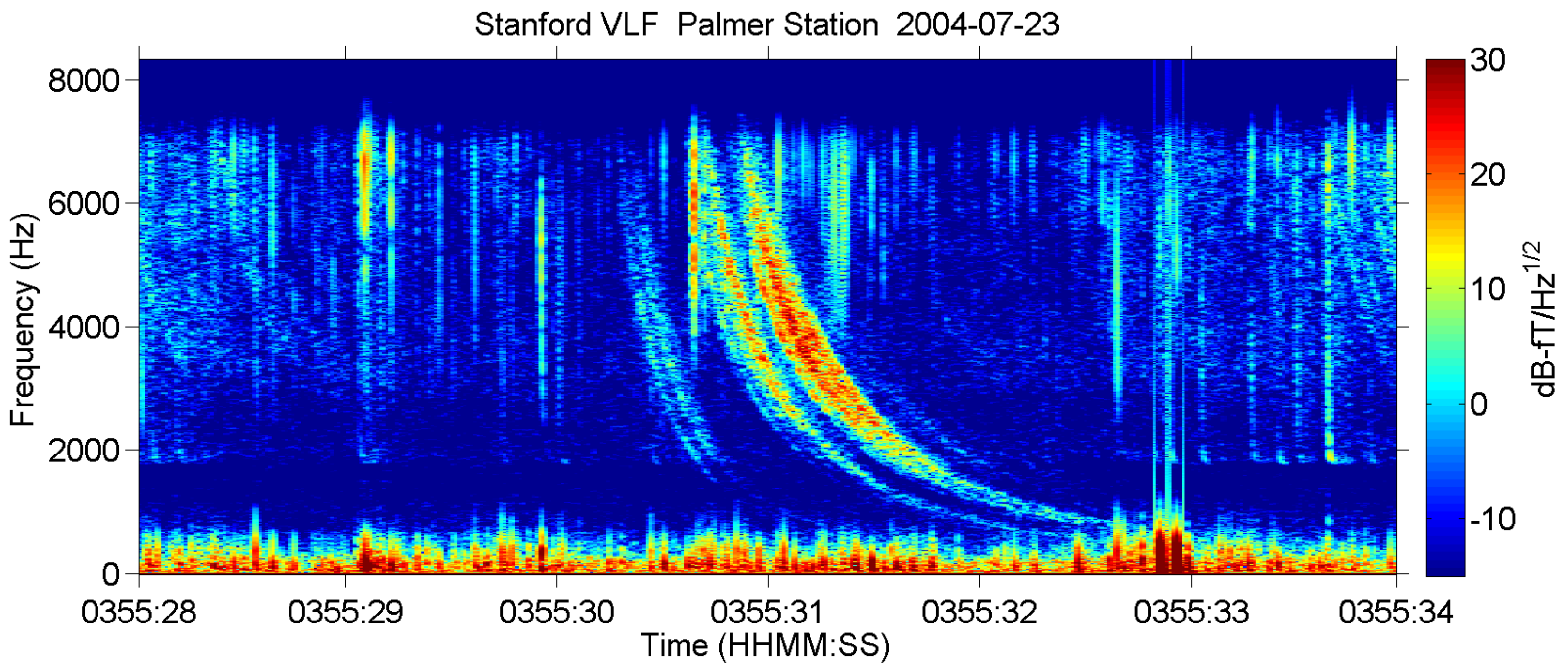
"whistler mode": v_g increases as ω increases

Nicholson, pp. 156-161

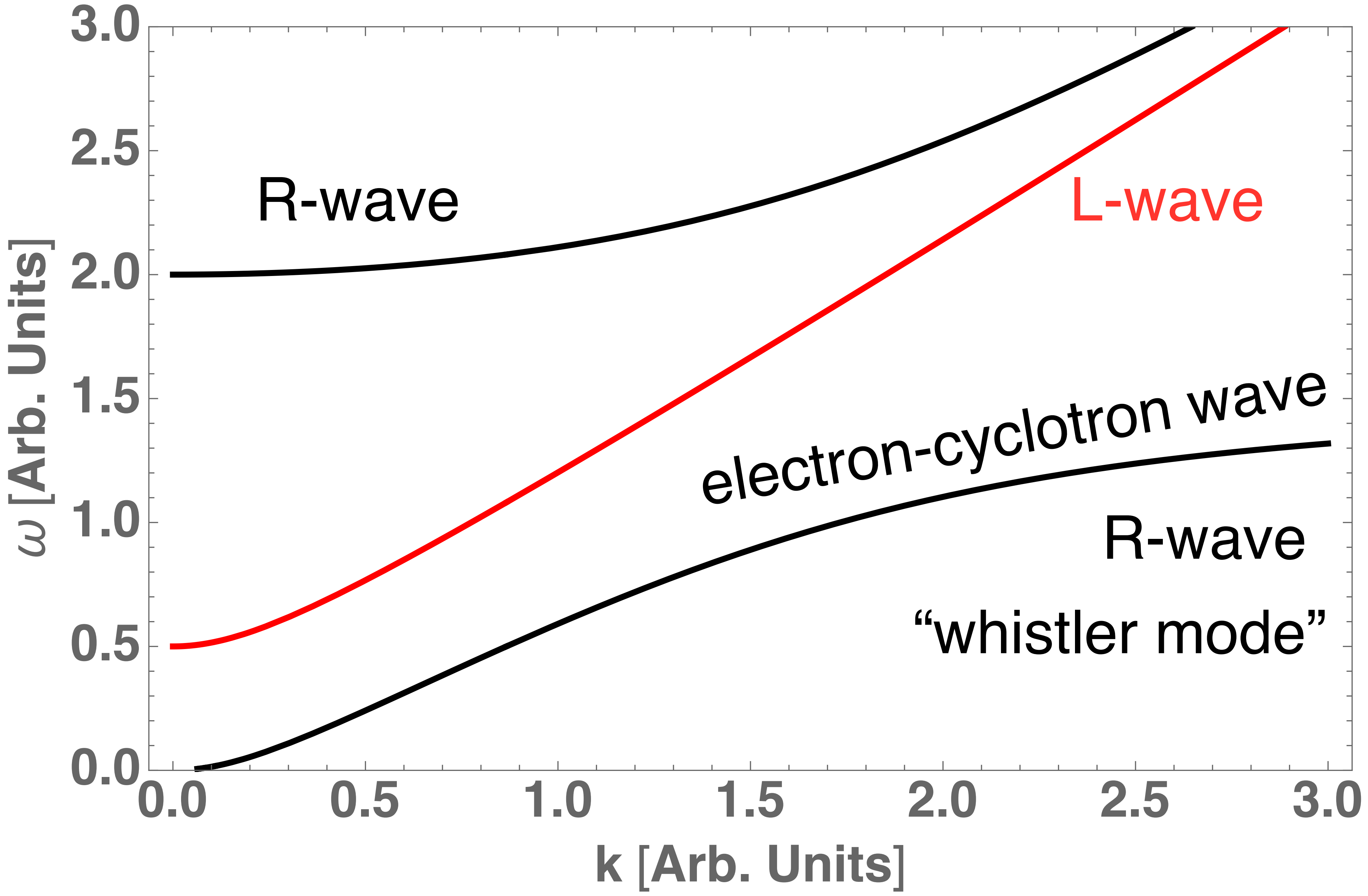
Propagation along B - whistler mode

Whistler

- E.g. lightning may create e.m. waves of various frequencies that may travel along different B field lines
- Single lightning can be at the origin of several whistlers
- High frequencies arrive first
- Listen to whistler using this link



$\omega_L < |\Omega_e|$



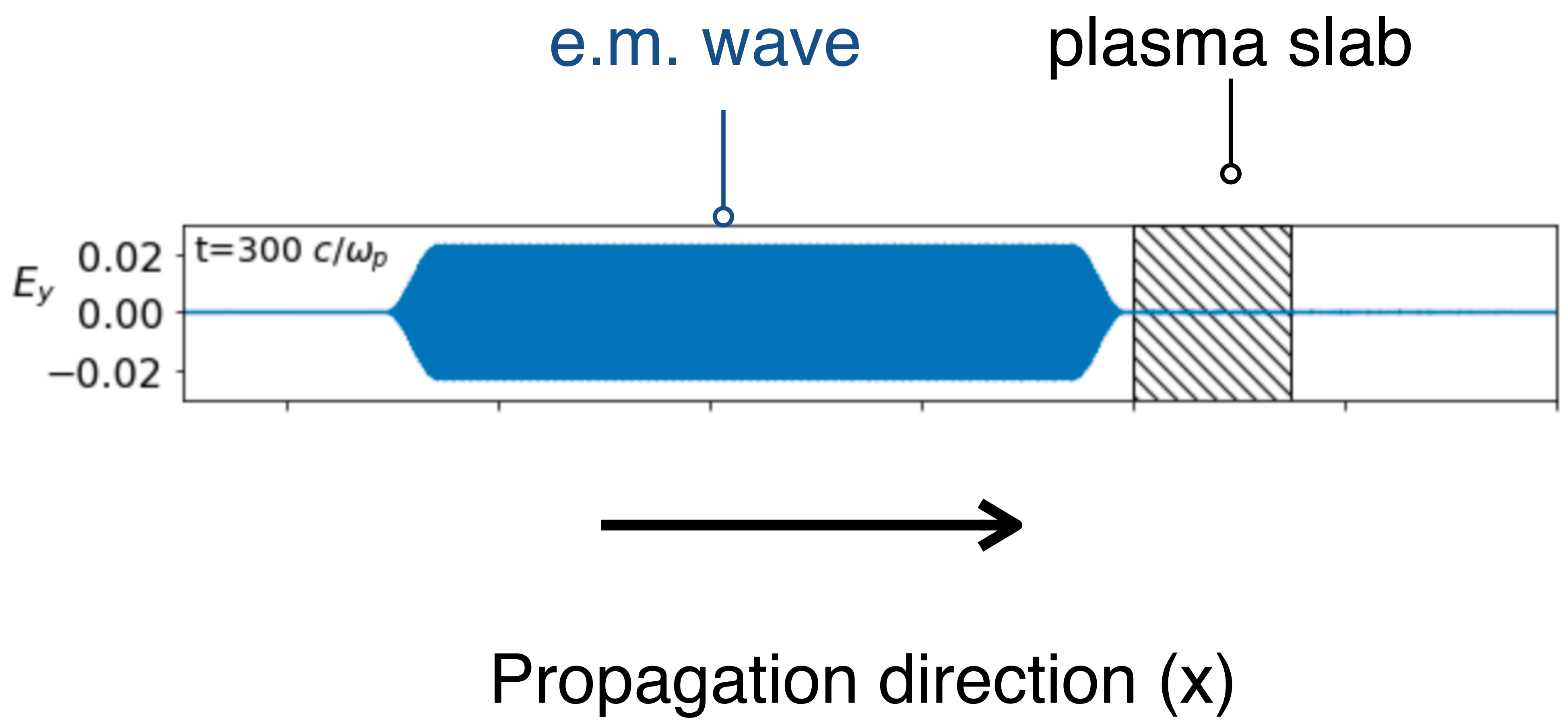
"whistler mode": v_g increases as ω increases

Nicholson, pp. 156-161

Numerical example - plasma e.m. wave polariser

Simulation parameters

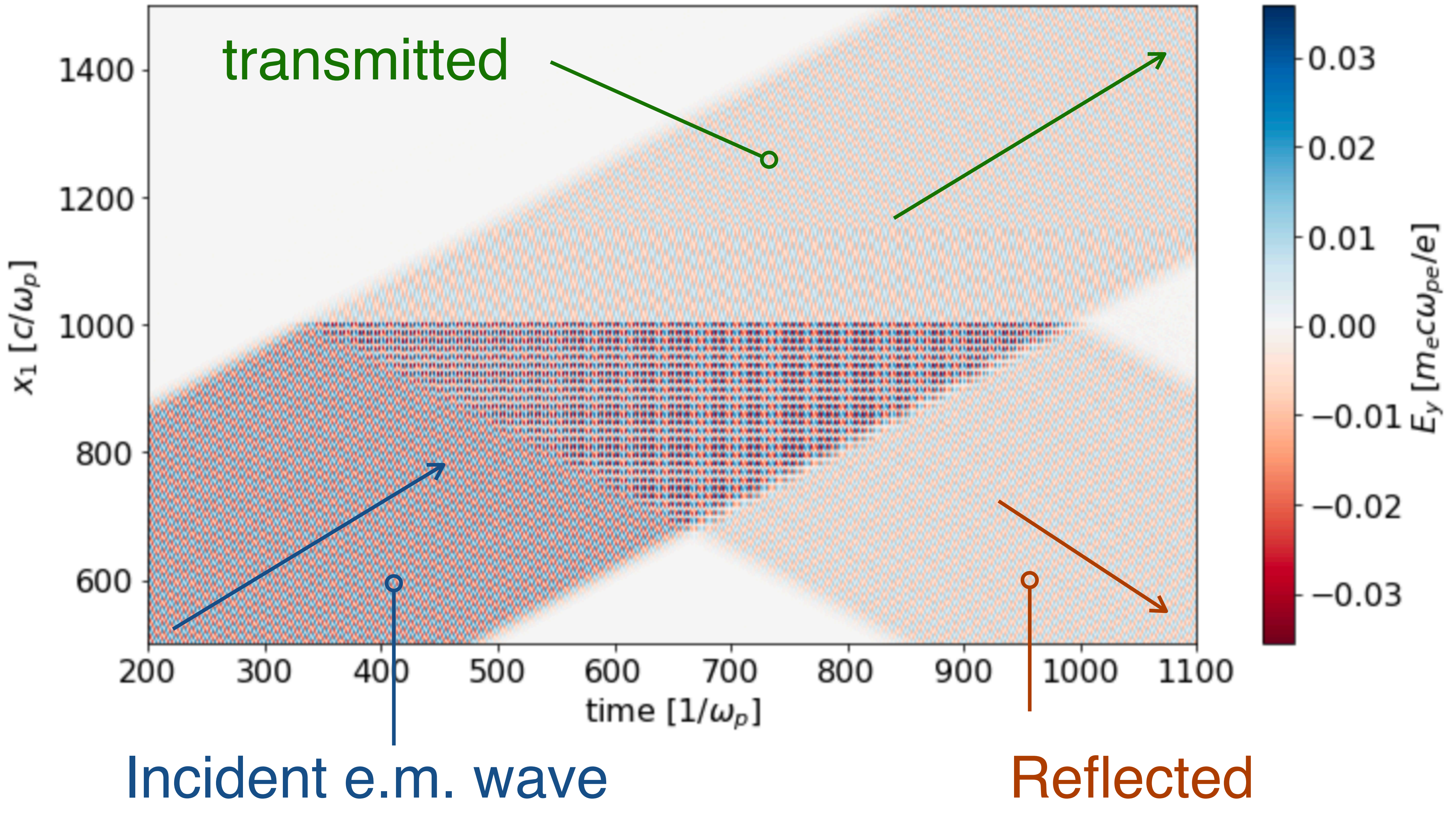
- Magnetised plasma with $\omega_c = 2\omega_p$
- External electromagnetic wavepacket ($\omega = 2.3\omega_p$) polarised along $\mathbf{e}_y$
- 'Thin' plasma slab



Results

Cut-offs: $\omega_R = 2.85\omega_p$ and $\omega_L = 0.35\omega_p$

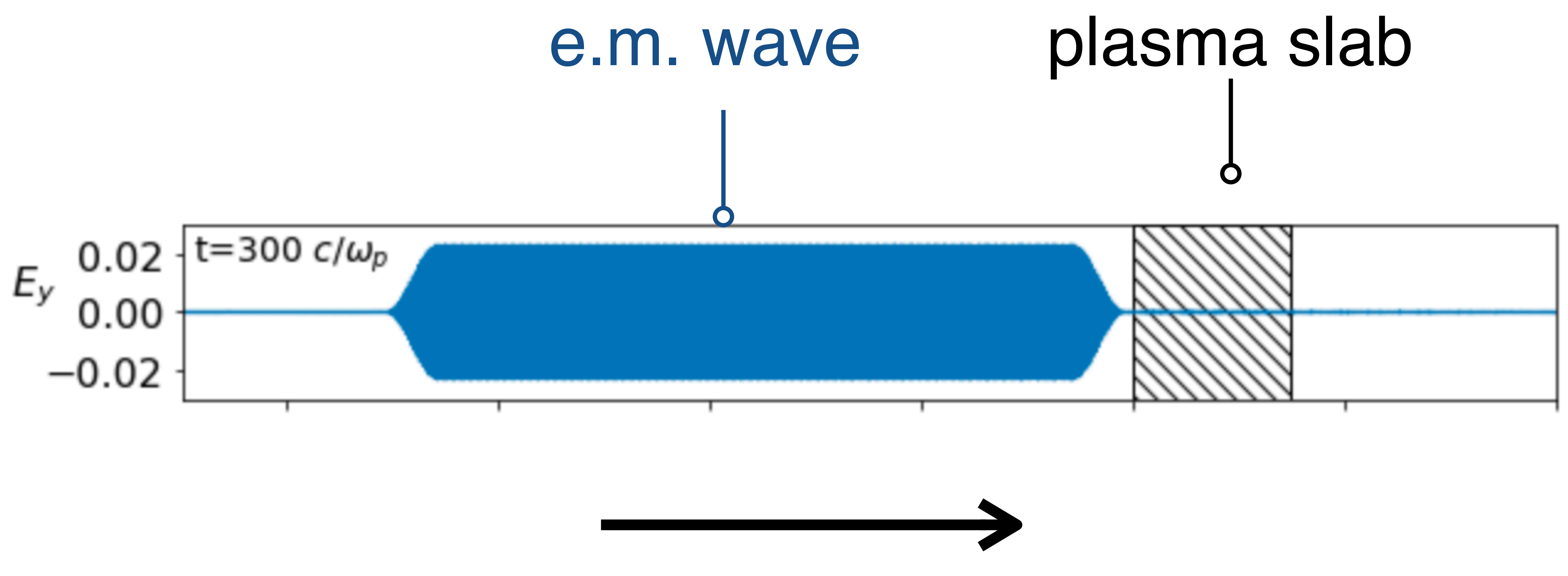
"Waterfall" plot $E_y(x, t)$



Numerical example - plasma e.m. wave polariser

Simulation parameters

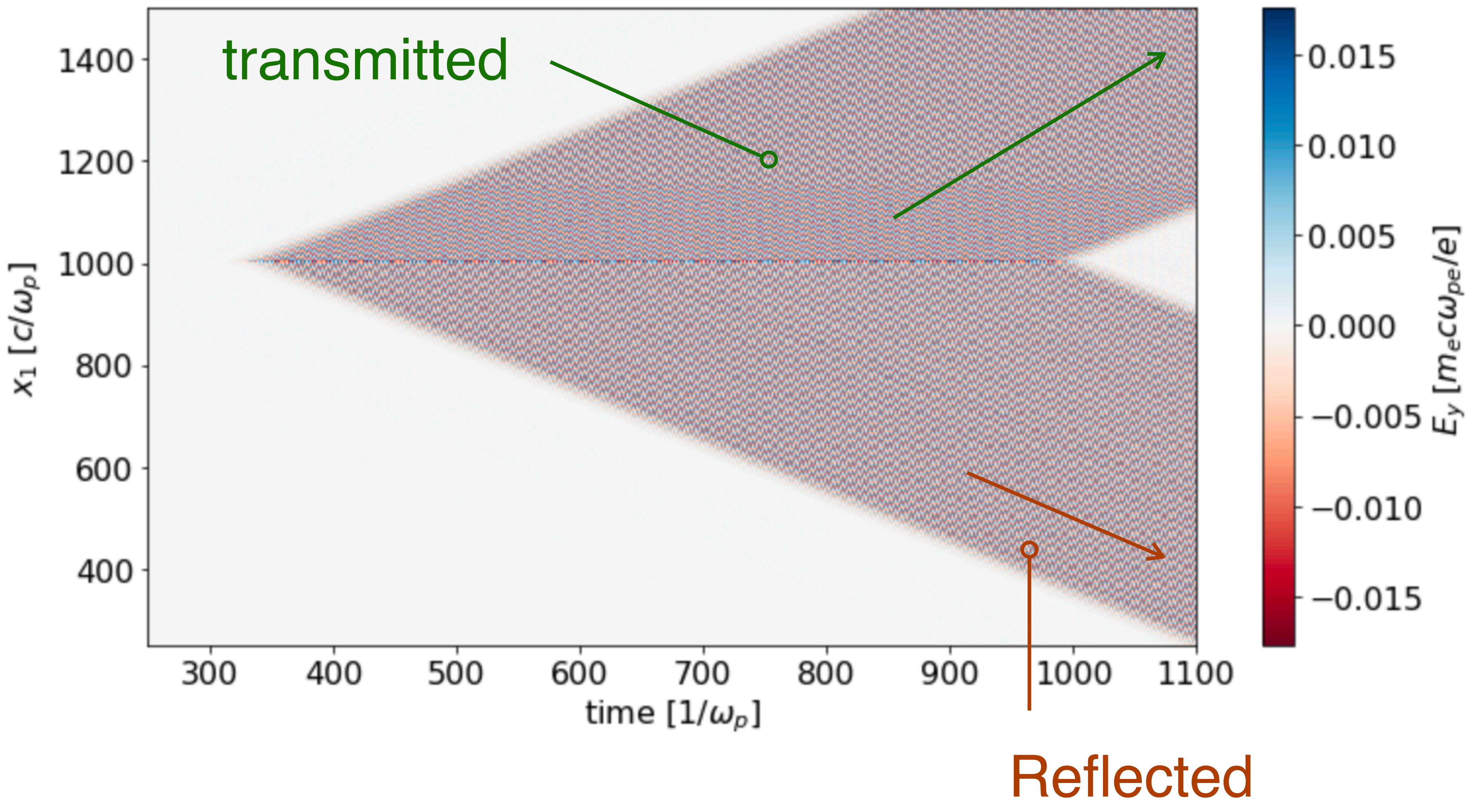
- Magnetised plasma with $\omega_c = 2\omega_p$
- External electromagnetic wavepacket ($\omega = 2.3\omega_p$) polarised along $\mathbf{e}_y$
- 'Thin' plasma slab



Results

Cut-offs: $\omega_R = 2.85\omega_p$ and $\omega_L = 0.35\omega_p$

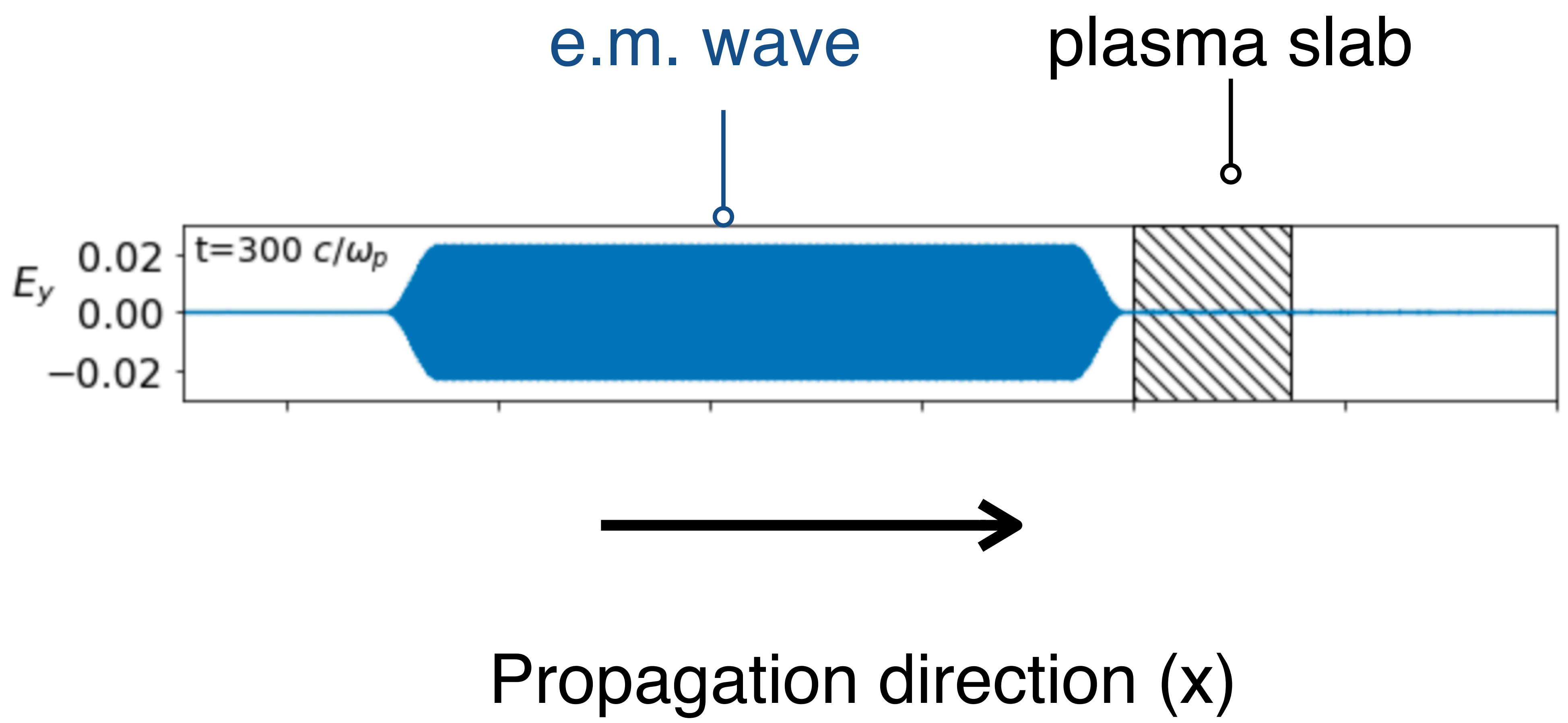
"Waterfall" plot $E_y(x, t)$



Numerical example - plasma e.m. wave polariser

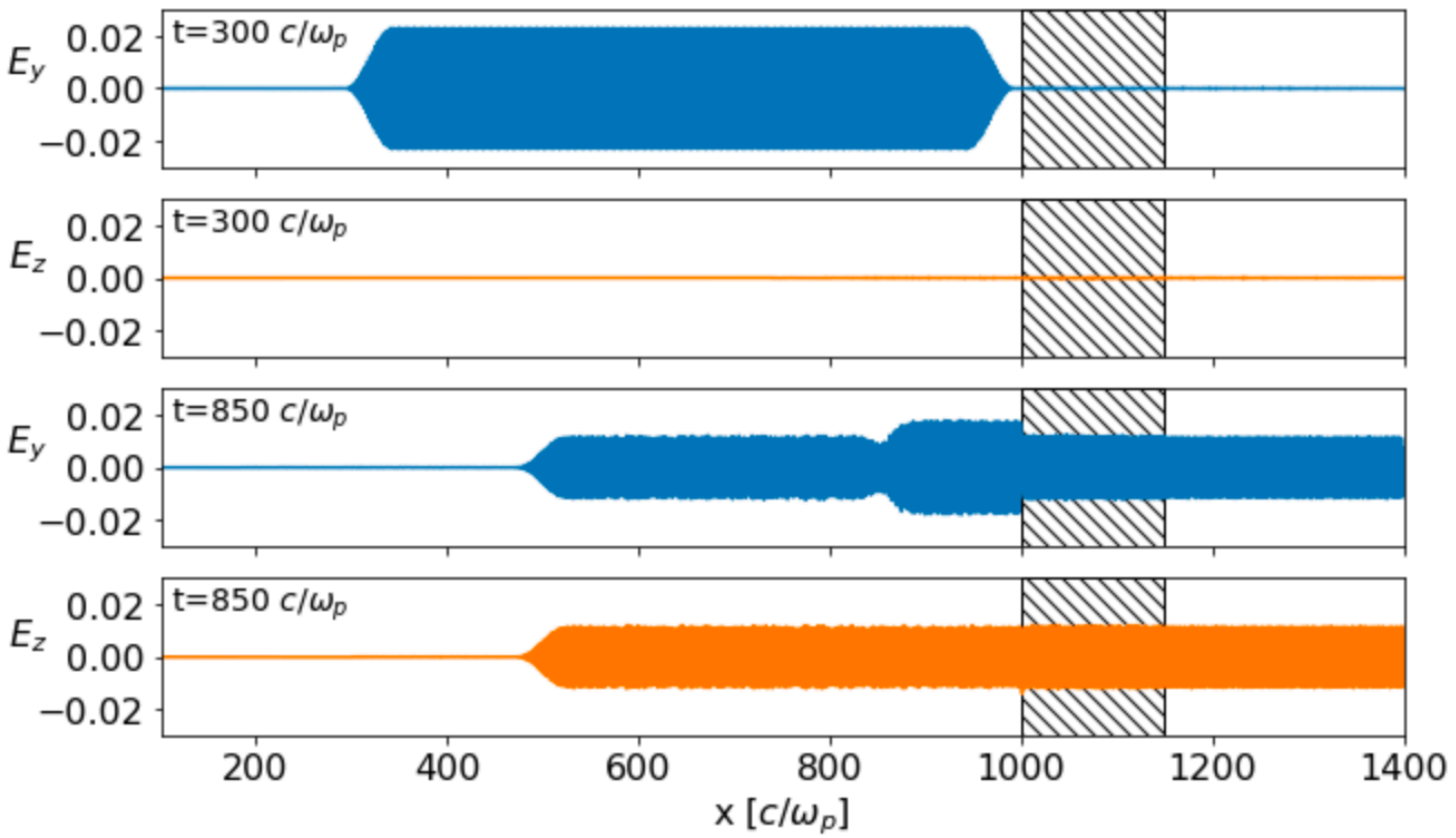
Simulation parameters

- Magnetised plasma with $\omega_c = 2\omega_p$
- External electromagnetic wavepacket ($\omega = 2.3\omega_p$) polarised along $\mathbf{e}_y$
- 'Thin' plasma slab



Field evolution snapshots

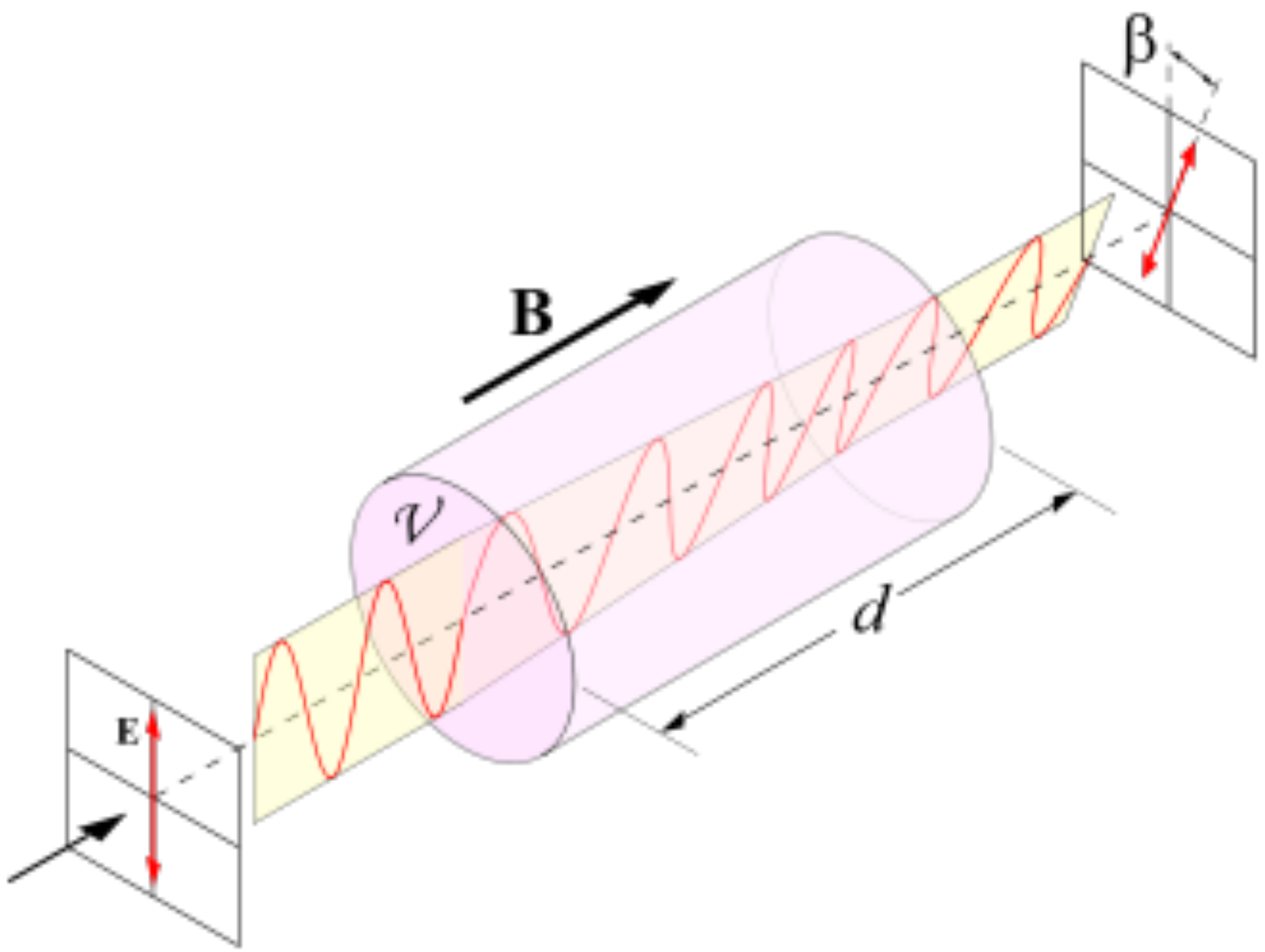
Incident linear polarisation



Transmitted circular polarisation!

Propagation along B - Faraday rotation

Configuration



High frequencies $\omega > \omega_R$

- Linearly polarised wave can be decomposed into left and right circularly polarised light
- Each component travels with different phase speeds in a magnetised plasma
- Rotation of the polarisation!
- Useful diagnostic for plasma density in astrophysics (estimates for ambient B fields known)!

Nicholson, p. 161