Plasma Physics and Technology
Single particle motion

TÉCNICO LISBOA

Mestrado Integrado em Engenharia Física Tecnológica

Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico

How to study plasmas

- **single particle motion**
  - simple but powerful analysis
  - enables to investigate key waves and instabilities in plasma physics
- plasma kinetic equations
  - general approach
  - can be solved using computer programs
- fluid equations
  - plasma waves and instabilities
  - interaction with electromagnetic waves

why studying single particle motions is important?

- interactions between individual pairs of charged particles is small
- fields can become large when many particles interact, and lead to plasma oscillations
- fields from many particles are smooth with small fluctuations on top
- small fluctuations lead to collisions
- macroscopic smoothed fields give rise to collective motions of the plasma
- if we can neglect collisions we can treat the plasma as a collection of charged particles each moving in smoothed fields (self-consistent + external)
- **much insights can be gained by analysing the single particle motion**
- further analysis (not to be covered in our course): **make fields self-consistent with particle motion**

energy equation

work done by electric field in a charged particle

- equation of motion (non-relativistic)

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = q\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right)$$

- dot product with $\mathbf{v}$

$$\left.\begin{array}{l} m\dfrac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\cdot\mathbf{v} = \dfrac{1}{2}m\dfrac{\mathrm{d}\mathbf{v}^2}{\mathrm{d}t} \\ \\ (\mathbf{v}\times\mathbf{B})\cdot\mathbf{v} = 0 \end{array}\right\} \quad \frac{1}{2}m\frac{\mathrm{d}\mathbf{v}^2}{\mathrm{d}t} = q\mathbf{E}\cdot\mathbf{v}$$

energy conservation law

- integration over particle orbit

$$\underbrace{\Delta\left(\frac{1}{2}m\mathbf{v}^2\right)}_{\text{kinetic energy gain}} = \int q\mathbf{E}\cdot\mathbf{v}\mathrm{d}t = \underbrace{\int q\mathbf{E}\cdot\mathrm{d}\mathbf{l}}_{\text{work done by electric field}}$$

- if $\mathbf{E} = -\nabla\Phi$

$$\underbrace{\left(\frac{1}{2}m\mathbf{v}^2\right) + q\Phi = \text{constant}}_{\text{conservation of kinetic plus potential energy}}$$

Dawson, pp 93-98

motion in uniform magnetic field - cyclotron motion

- velocity component along magnetic field does not change ($v_{\parallel} = \text{constant}$)

- magnitude of velocity component perpendicular to magnetic field does not change, as just showed ($\mathbf{v}_{\perp}^2 = \text{constant}$)

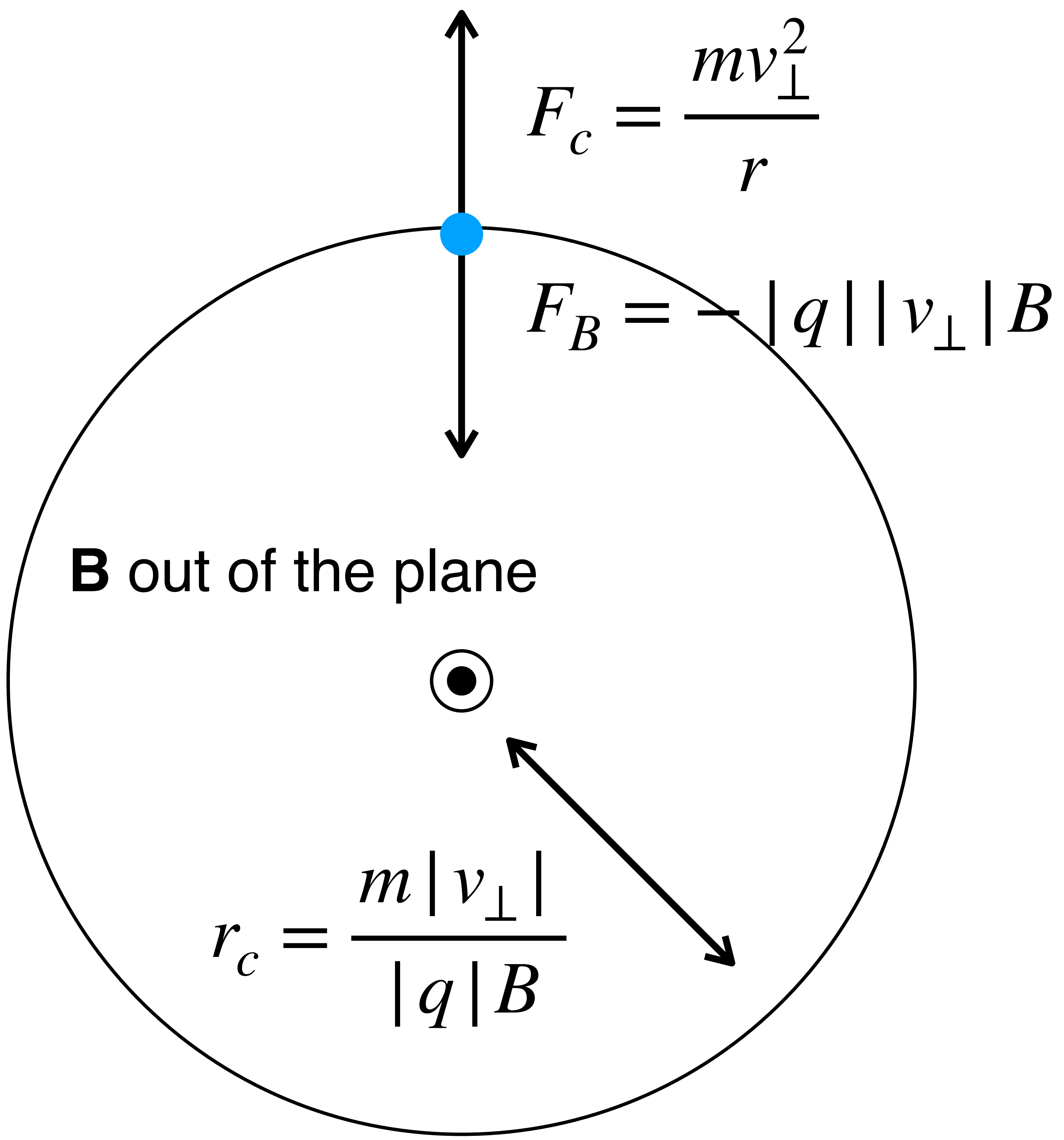
- motion is circular about the magnetic field

- radius of circular orbit

$$m\frac{v_{\perp}^2}{r_c} = |q||v_{\perp}|B \Leftrightarrow r_c = \frac{m|\mathrm{v}_{\perp}|}{|q|B}$$

- angular frequency

$$\omega_c = \frac{|q|B}{m}$$



Dawson, pp 93-98

cyclotron motion - formal derivation

equation of motion

- assumptions
  - **B** is along z
  - **v** is along x and y
- from equation of motion

$$\frac{\mathrm{d}v_x}{\mathrm{d}t} = \omega_c v_y$$

$$\frac{\mathrm{d}v_y}{\mathrm{d}t} = -\omega_c v_x$$

$$\omega_c = \frac{qB_z}{m}$$

integration of equations of motion

- defining $w = v_x + i v_y$
- leads to $\frac{\mathrm{d}w}{\mathrm{d}t} = -i\omega_c w$
- integration yields

$$w = w_0 \exp\left(-i\omega_c t\right) \Rightarrow z = x + iy = i\frac{w_0}{\omega_c}\exp\left(-i\omega_c t\right) + z_0$$

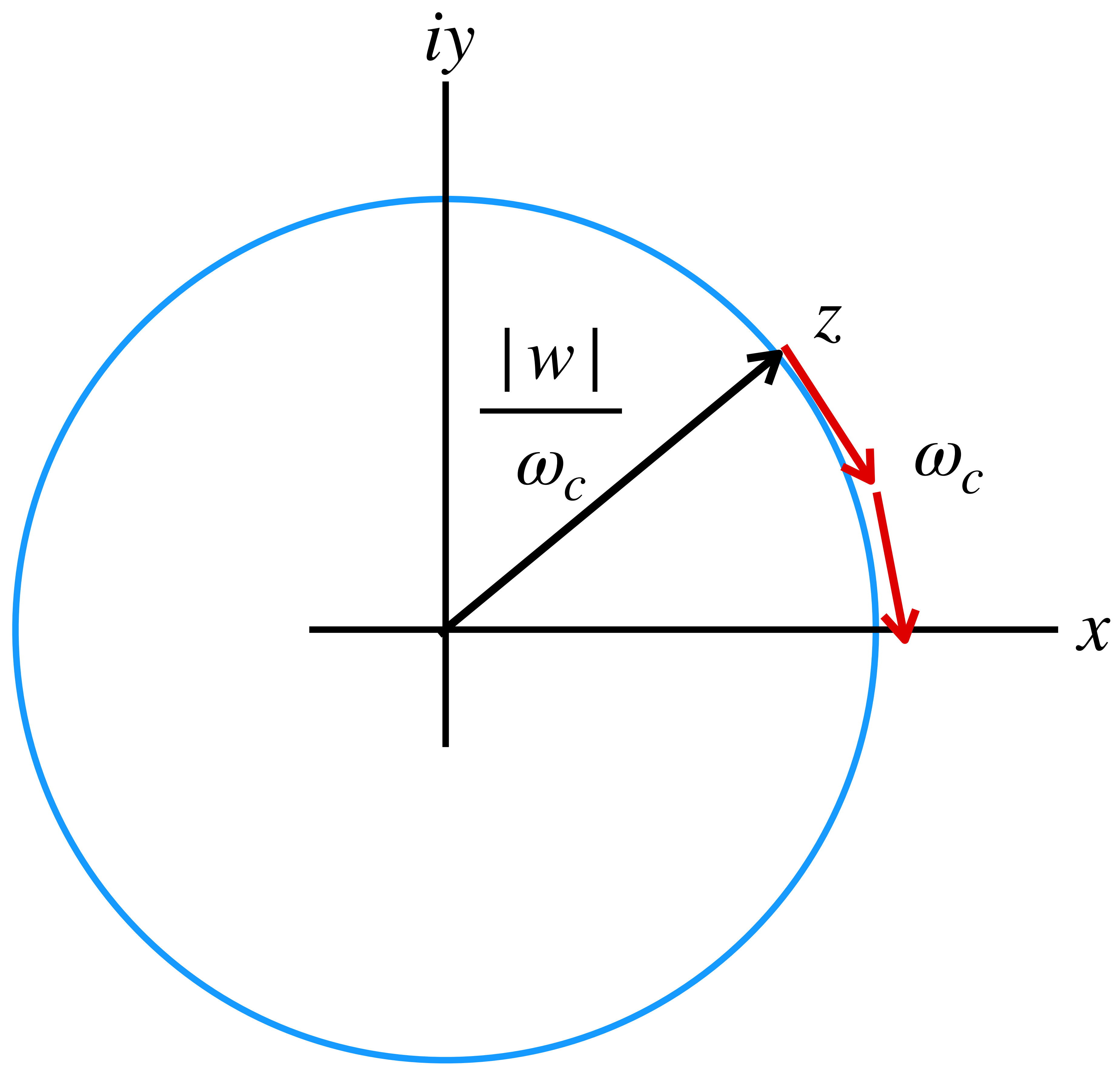
- where

$$z_0 = x_0 + iy_0 - i\frac{w_0}{\omega_c}$$

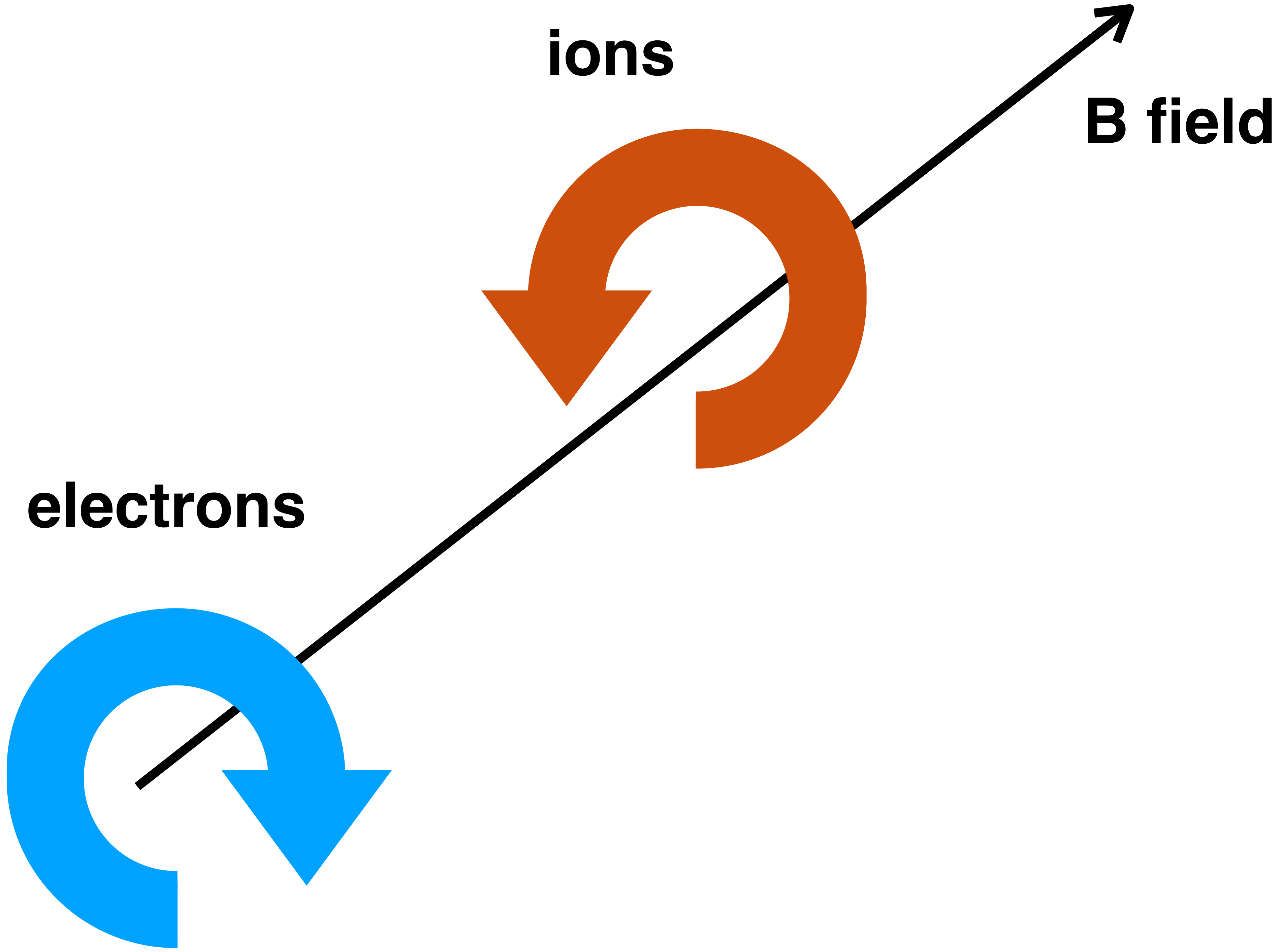
Dawson, pp 93-98

cyclotron motion - formal derivation

motion in complex plane



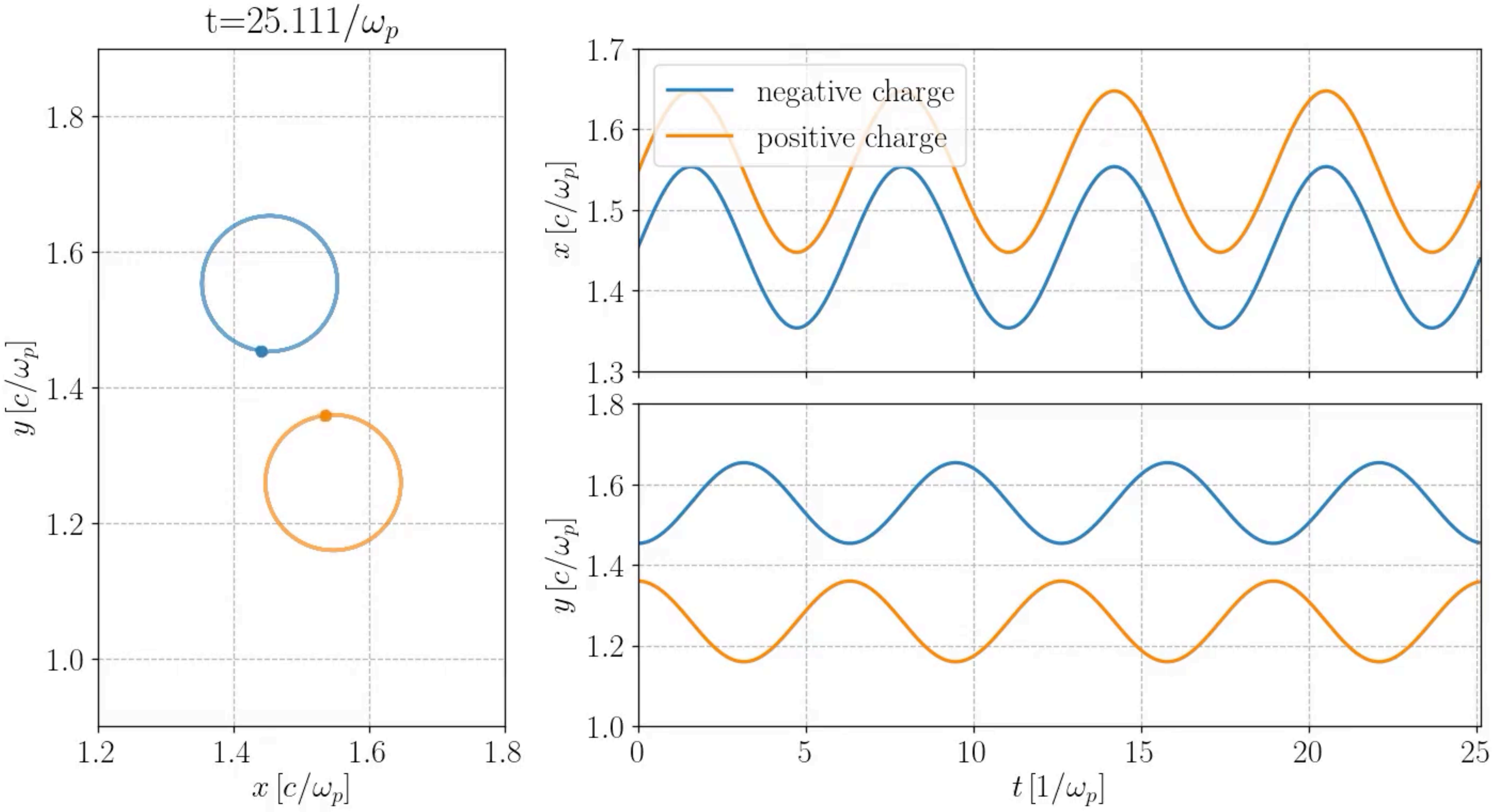
direction of rotation depends on charge



Dawson, pp 93-98

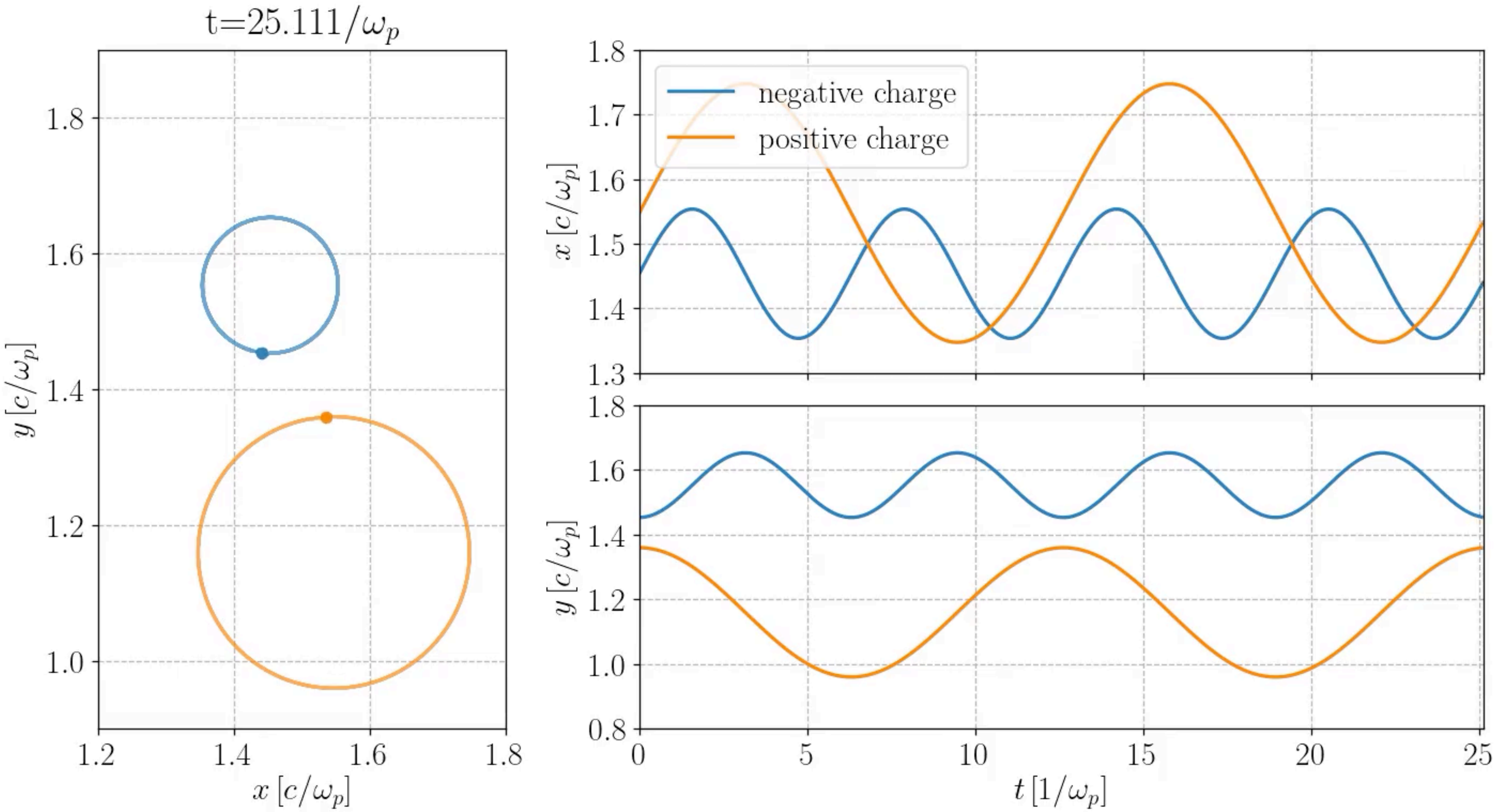
cyclotron motion - simulation

B field out of the plane; both particles have the same mass



Dawson, pp 93-98

B field out of the plane; positive charge is more massive than negative charge



plasma is a diamagnetic medium:

the motion of the charges produces a current that induces a magnetic field into the plane

Dawson, pp 93-98

magnetic moment

- a current loop (I is the average current, A is the area of the loop) has a dipole moment

$$\mu = IA$$

- average current is the average charge per unit time which passes a point in the orbit

$$I = \frac{q}{\tau} = \frac{q\omega_c}{2\pi}$$

- area of the loop is

$$A = \pi r_c^2 = \pi \frac{m^2 \mathbf{v}_\perp^2}{q^2 \mathbf{B}^2}$$

- thus

$$\mu = \frac{m\mathbf{v}_\perp^2}{2|\mathbf{B}|} = \frac{W_\perp}{|\mathbf{B}|}$$

or

$$\boldsymbol{\mu} = -\frac{W_\perp}{\mathbf{B}^2}\mathbf{B}$$

Dawson, p 98

electric field drift (or ExB drift) - derivation

constant and spatially uniform E and B fields. motion along the magnetic field

- equation of motion (non-relativistic)

$$m\frac{\mathrm{d}v_{\|}}{\mathrm{d}t} = qE_{\|}$$

- solution

$$v_{\|} = \frac{qE_{\|}}{m}t + v_{\|0} \qquad x_{\|} = \frac{qE_{\|}}{m}\frac{t^2}{2} + v_{\|0}t + x_{\|0}$$

motion across magnetic field

- equation of motion

$$m\frac{\mathrm{d}v_{\perp}}{\mathrm{d}t} = q\left(\mathbf{E}_{\perp} + \underbrace{\mathbf{v}_{\perp}\times\mathbf{B}}\right)$$

- electric and magnetic components perpendicular to B
- it is possible to balance both components

balance electric and magnetic forces

$$m\frac{\mathrm{d}v_{\perp}}{\mathrm{d}t} = \mathbf{E}_{\perp} + \mathbf{v}_{\perp}\times\mathbf{B} = 0$$

- particle travels with constant perpendicular velocity.
- cross product with $\mathbf{B}$ gives

$$\mathbf{v}_{\perp} \equiv \mathbf{v}_E = \frac{\mathbf{E}_{\perp}\times\mathbf{B}}{\mathbf{B}^2}$$

full solution

- writing

$$\mathbf{v}_{\perp} = \mathbf{v}_1 + \mathbf{v}_E$$

- gives for $\mathbf{v}_1$

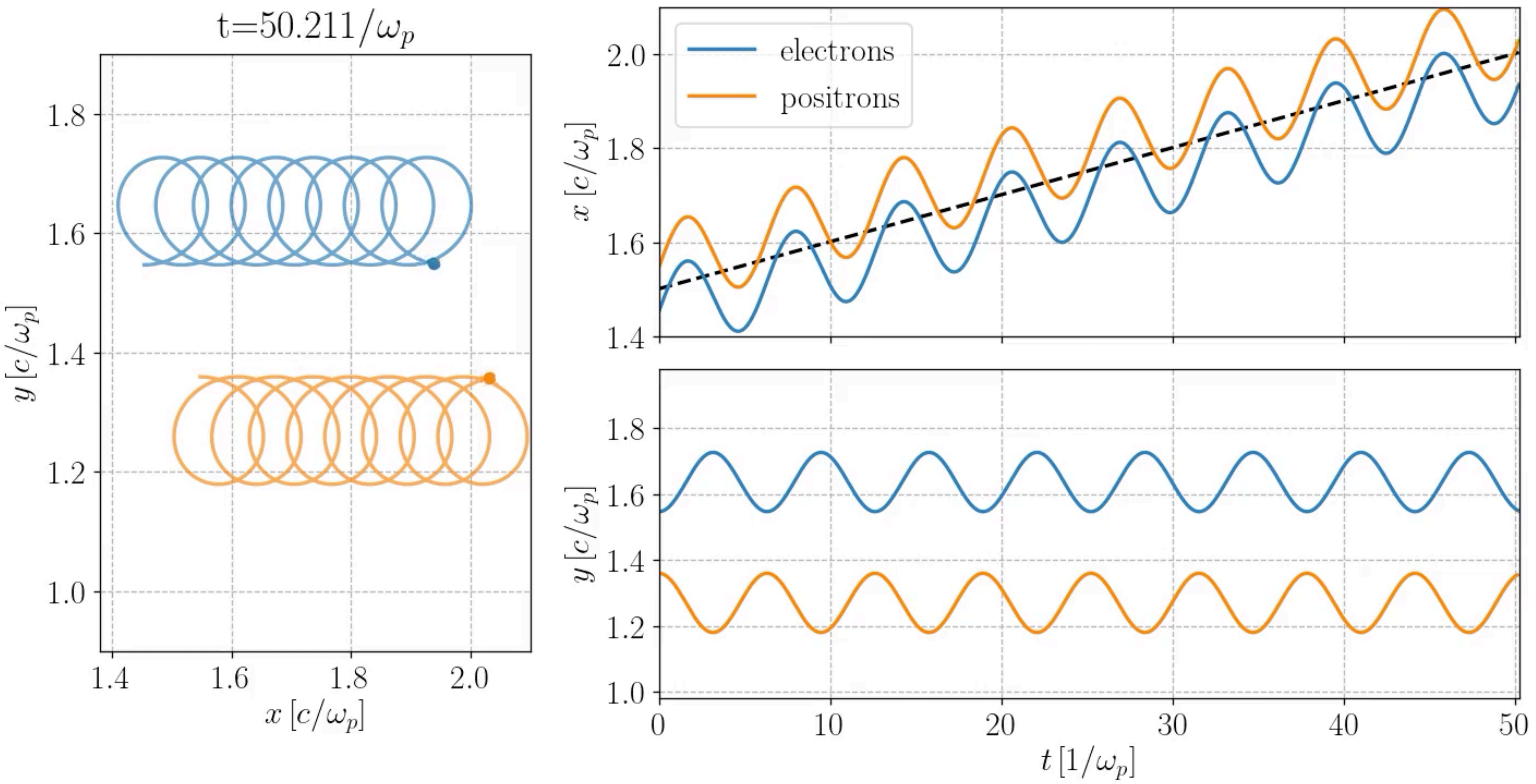
$$m\frac{\mathrm{d}\mathbf{v}_1}{\mathrm{d}t} = q\mathbf{v}_1\times\mathbf{B}$$

- **cyclotron motion plus a drift velocity**

Dawson, p 102-106

electric field drift (or ExB drift) - simulation

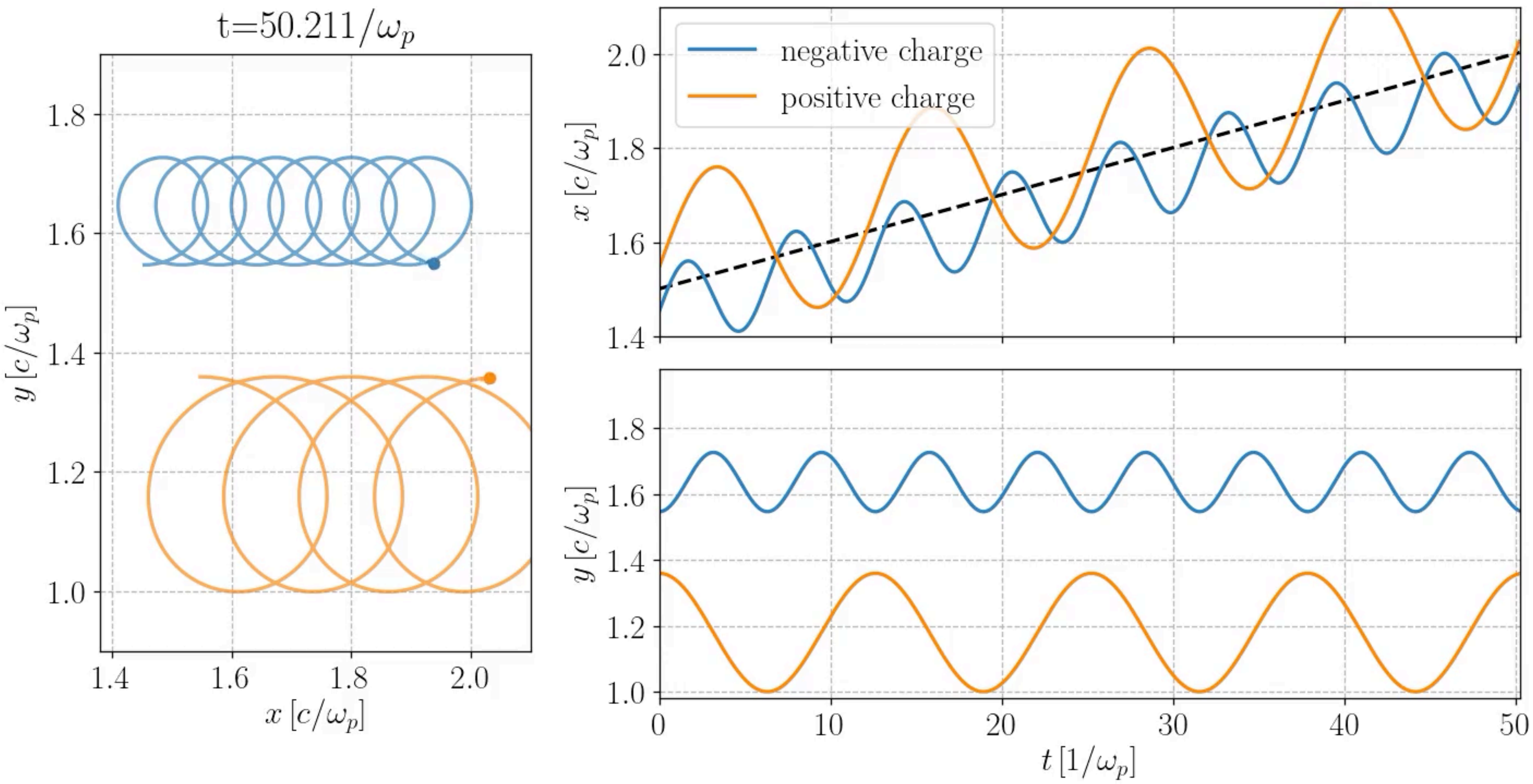
B field out of the plane; E in the y direction; both particles have the same mass



Dawson, p 102-106

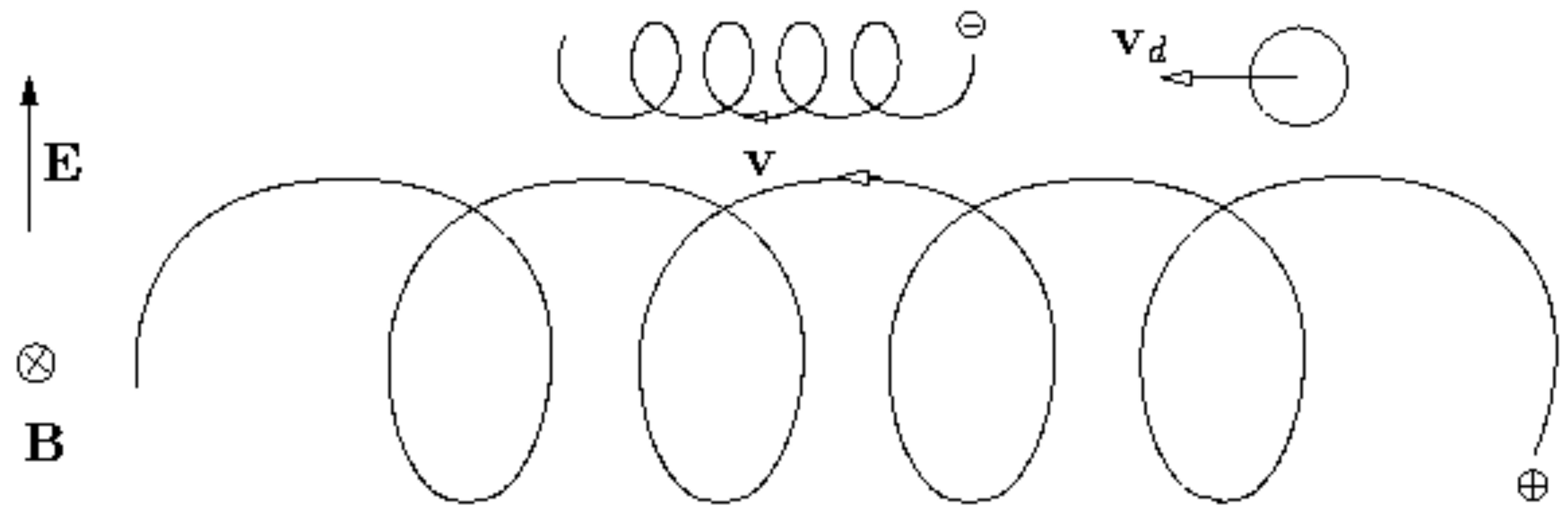
electric field drift (or ExB drift) - simulation

B field out of the plane; E in the y direction; positive charge is more massive than negative charge



Dawson, p 102-106

electric field drift (or ExB drift) - physical picture



- as a positive charge spirals in the magnetic field its energy changes due to the electric field
- the charge moves faster in the upper part of the trajectory
- curvature is smaller in the upper part of the trajectory than in the lower part and the particle drifts
- a negative charge spirals in the opposite direction
- radius of curvature is larger in the bottom part of the trajectory. drift is in the same direction

Dawson, p 102-106

ExB drift and the *de Hoffman-Teller* frame

moving across a magnetic field gives rise to an electric field:

$$\mathbf{E}'_\perp = \gamma\left(\mathbf{E}_\perp + \mathbf{v}\times\mathbf{B}\right)$$

$$= \gamma\left(\mathbf{E}_\perp + \frac{\mathbf{E}_\perp\times\mathbf{B}}{\mathbf{B}^2}\times\mathbf{B}\right)$$

$$= \gamma\left(\mathbf{E}_\perp - \mathbf{E}_\perp\frac{\mathbf{B}^2}{\mathbf{B}^2}\right) = 0$$

For $\mathbf{v} = \mathbf{v}_E$ the equation above is zero, i.e. there is no electric field when moving with the drift velocity. Particles see a magnetic field and move accordingly.

Dawson, p 102-106

electric field drift (or ExB drift) - significant features

- in a neutral plasma, since positive and negative charges drift in the same direction, **the ExB drift generates no current**

- **no work is done on average on either (positively or negatively charged) particle** because the electric field is perpendicular to the drift velocity

Dawson, p 102-106

external force (e.g. gravity) drift

derivation

- equation of motion (non-relativistic)

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{F} + q\mathbf{v}\times\mathbf{B}$$

- make substitution $\mathbf{E} = \mathbf{F}/q$

$$\mathbf{v}_F = \frac{1}{q}\frac{\mathbf{F}\times\mathbf{B}}{\mathbf{B}^2}$$

- **motion is the cyclotron trajectory plus the drift**

important features

- drift depends on the sign of the charge of the particle
- an external force acting on a neutral cloud will cause particles with opposite charges to drift in opposite directions.
- the drift motion now gives rise to a current in the plasma

Dawson, p 106-107

time-varying electric field - polarisation drift

initial assumptions

- spatially uniform E and B fields.
- the direction of E is constant in space but magnitude varies in time
- consider $\mathbf{E}\cdot\mathbf{B}=0$
- electric field varies little in a Larmor rotation $\frac{1}{\mathbf{E}}\frac{1}{\omega_c}\frac{\mathrm{d}\mathbf{E}}{\mathrm{d}t}\ll 1$

derivation on physical grounds: kinetic energy change

- to first approximation:

$$\mathbf{v}_\perp=\mathbf{v}_{\mathrm{Larmor}}+\frac{\mathbf{E}(t)\times\mathbf{B}}{\mathbf{B}^2}$$

- kinetic energy change averaged over the Larmor period

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m}{2}\mathbf{v}_\perp^2\right)=m\mathbf{v}_E\frac{\mathrm{d}\mathbf{v}_E}{\mathrm{d}t}=\frac{m}{\mathbf{B}^2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathbf{E}^2}{2}\right)$$

derivation on physical grounds: energy balance

- energy must be supplied by E field

$$qv_{\parallel E}\mathbf{E}=\frac{m}{\mathbf{B}^2}\mathbf{E}\frac{\mathrm{d}\mathbf{E}}{\mathrm{d}t}\Rightarrow v_{\parallel E}=\frac{m}{q\mathbf{B}^2}\frac{\mathrm{d}\mathbf{E}}{\mathrm{d}t}$$

Dawson, pp 107-110

time-varying electric field - polarisation drift

derivation on physical grounds: inertial forces

- ExB drift changes with time. Acceleration is:

$$\mathbf{a} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathbf{E}(t)\times\mathbf{B}}{\mathbf{B}^2}\right)$$

- Force in the guiding centre frame is $\mathbf{F} = -m\mathbf{a}$

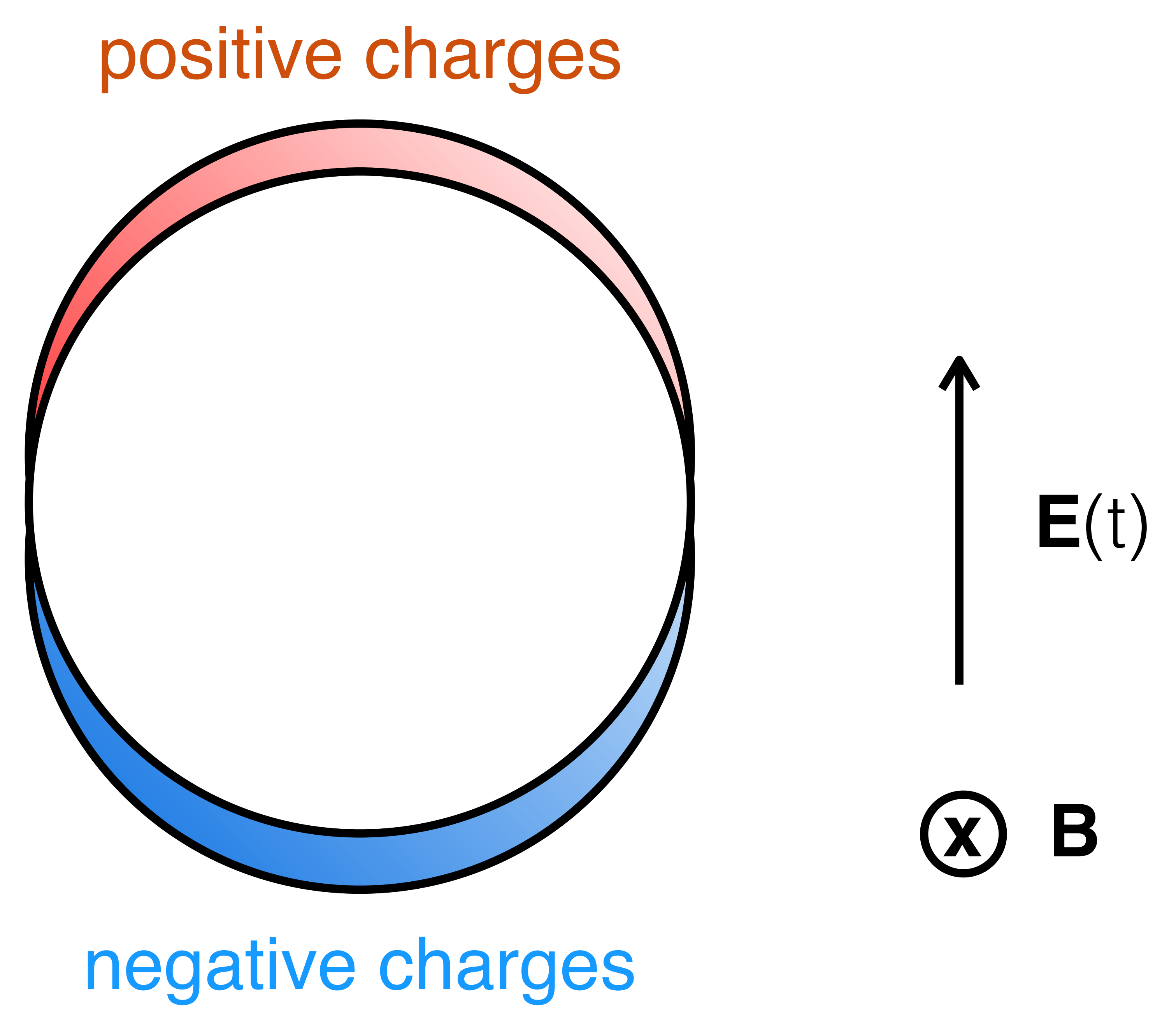
$$\mathbf{F} = -m\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathbf{E}(t)\times\mathbf{B}}{\mathbf{B}^2}\right)$$

- Treat $\mathbf{F}$ as an external force which induces a drift:

$$\mathbf{v}_{\|E} = \mathbf{v}_{DE} = \frac{m}{q\mathbf{B}^2}\frac{\mathrm{d}\mathbf{E}}{\mathrm{d}t}$$

charge distribution due to polarisation drift

positive charges move in direction of **E.**
negative charges move in the opposite to **E**



Dawson, pp 107-110

time-varying magnetic field - magnetic moment

initial assumptions

- spatially uniform B field with amplitude varying in time
- there will be an E field set up because B varies with time
- this will give rise to ExB and polarisation drifts
- **we are interested in the fact that E has a curl and will hence do work on a circulating charge**
- lets imagine that we subtract the ExB drift

work done by electric field in a closed orbit

- charge perpendicular energy of the particle

$$\delta W_\perp = q \int \mathbf{E} \cdot \mathrm{d}l$$

- around a closed orbit

$$\oint \mathbf{E} \cdot \mathrm{d}l = \int \nabla \times \mathbf{E} \cdot \mathrm{d}\mathbf{A} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot \mathrm{d}\mathbf{A}$$

flux of $\partial \mathbf{B}/\partial t$ through the orbit

- since $\partial \mathbf{B}/\partial t$ is essentially constant inside the orbit

$$-\int \frac{\partial \mathbf{B}}{\partial t} \cdot \mathrm{d}\mathbf{A} = (\pm)\pi r_c^2 \frac{\partial \mathbf{B}}{\partial t}$$

- we only need the plus sign because **dA** is always antiparallel to **B**

Dawson, pp 114-116

time-varying magnetic field - magnetic moment

initial assumptions

- spatially uniform B field with amplitude varying in time
- there will be an E field set up because B varies with time
- this will give rise to ExB and polarisation drifts
- **we are interested in the fact that E has a curl and will hence do work on a circulating charge**
- lets imagine that we subtract the ExB drift

work done by electric field in a closed orbit

- charge perpendicular energy of the particle

$$\delta W_\perp = |q| \pi r_c^2 \frac{\partial B_z}{\partial t}$$

- or

$$\delta W_\perp = W_\perp \frac{2\pi}{\omega_c B} \frac{\partial B_z}{\partial t}$$

conservation of magnetic moment

- for a single period

$$\frac{\delta W_\perp}{W_\perp} = \frac{\delta B_z}{B_z} \Leftrightarrow \delta\left(\frac{W_\perp}{B_z}\right) = 0$$

- this means that

$$\frac{W_\perp}{B_z} = \mu = \text{constant}$$

Dawson, pp 114-116

drifts in non-uniform magnetic fields

- consider slow variations of the magnetic field within the cyclotron radius

$$\frac{|\nabla \mathbf{B}| r_c}{|\mathbf{B}|} \ll 1$$

- general approach consists in finding the particle motion as a perturbation from the spatially uniform case

$$\mathbf{B}(\mathbf{r} + \boldsymbol{\rho}) = \mathbf{B}(\mathbf{r}) + \rho \cdot \nabla \mathbf{B}(\mathbf{r})$$

- Taylor expand the magnetic field about some $\mathbf{r}$ (which, in general, might depend on time)

$\nabla \mathbf{B}$ tensor

$$\begin{bmatrix} \frac{\partial B_x}{\partial x} & \frac{\partial B_x}{\partial y} & \frac{\partial B_x}{\partial z} \\ \frac{\partial B_y}{\partial x} & \frac{\partial B_y}{\partial y} & \frac{\partial B_y}{\partial z} \\ \frac{\partial B_z}{\partial x} & \frac{\partial B_z}{\partial y} & \frac{\partial B_z}{\partial z} \end{bmatrix}$$

Dawson, pp 116-117

non-uniform magnetic fields - diagonal terms

- these terms are not all independent since $\nabla \cdot \mathbf{B} = 0$
- coordinate system where $\mathbf{r} = \mathbf{0}$ and $\mathbf{B}(\mathbf{0}) = B_0 \mathbf{e}_z$
- neglect all off diagonal terms. then:

$$\begin{cases} B_z = B_0 + \left(\dfrac{\partial \mathbf{B}_z}{\partial z}\right)_0 z \\ B_y = \left(\dfrac{\partial \mathbf{B}_y}{\partial y}\right)_0 y \\ B_x = \left(\dfrac{\partial \mathbf{B}_x}{\partial x}\right)_0 x \end{cases}$$

$\nabla \mathbf{B}$ tensor

$$\begin{bmatrix} \dfrac{\partial B_x}{\partial x} & \dfrac{\partial B_x}{\partial y} & \dfrac{\partial B_x}{\partial z} \\ \dfrac{\partial B_y}{\partial x} & \dfrac{\partial B_y}{\partial y} & \dfrac{\partial B_y}{\partial z} \\ \dfrac{\partial B_z}{\partial x} & \dfrac{\partial B_z}{\partial y} & \dfrac{\partial B_z}{\partial z} \end{bmatrix}$$

Dawson, pp 117-123

(div)(conv)erging lines of force: force on a magnetic dipole

- because $\nabla \cdot \mathbf{B} = 0$ we can write

$$\frac{\partial B_z}{\partial z} = -\left[\left(\frac{\partial B_x}{\partial x}\right)_0 + \left(\frac{\partial B_y}{\partial y}\right)_0\right]$$

- or

$$B_z = B_0 - \left[\left(\frac{\partial B_x}{\partial x}\right)_0 + \left(\frac{\partial B_y}{\partial y}\right)_0\right] z$$

- consider particle moving in z along with the field lines. particle sees B field varying in time

- the temporal variation will be slow as long as the spatial variation is slowly varying in space (assume that particle is not very fast)

conservation of magnetic moment

energy conservation

- conservation of magnetic moment:

$$W_\perp = |B|\mu = |B|\frac{W_{\perp 0}}{B_0}$$

- since magnetic field does no work

$$W_\parallel = W - |B|\mu = W_{\parallel 0} + W_{\perp 0} - |B|\mu$$

- parallel energy of particle **must change**

- conservation of energy differential form is the force on a magnetic dipole (minus sign is because the dipole is diamagnetic)

$$m\frac{\mathrm{d}v_\parallel}{\mathrm{d}t} = -\mu\frac{\mathrm{d}|B|}{\mathrm{d}z}$$

Dawson, pp 117-123

example: magnetic mirror

- recall the expression for the longitudinal energy

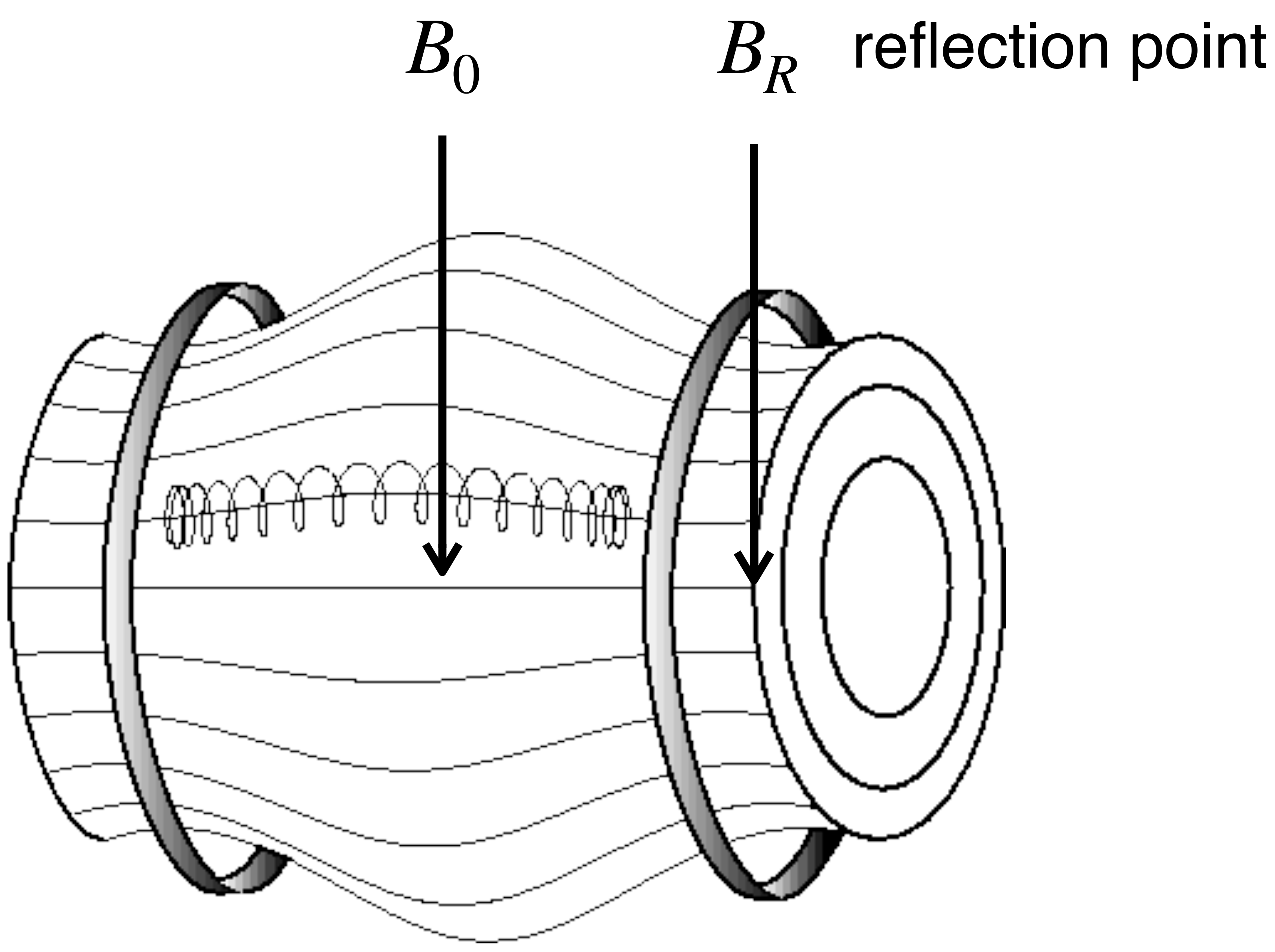
$$W_{||} = W - |B|\mu = W_{||0} + W_{\perp 0} - |B|\mu$$

- when |B| increases $W_{||}$ decreases and vice versa.

- thus charged particles traveling along z can be reflected due to the convergence of magnetic field lines which increases B close to the edges

- Conservation of energy + conservation of magnetic moment give the critical value for the magnetic field ratio to ensure reflection as a function of the particle transverse velocity:

$$\frac{B_0}{B_R} = \frac{\mathbf{v}_{\perp}^2}{\mathbf{v}_{\perp 0}^2 + \mathbf{v}_{||0}^2}$$



example: magnetic mirror

- the magnetic field for reflection is higher for smaller transverse velocities

- defining the pitch angle

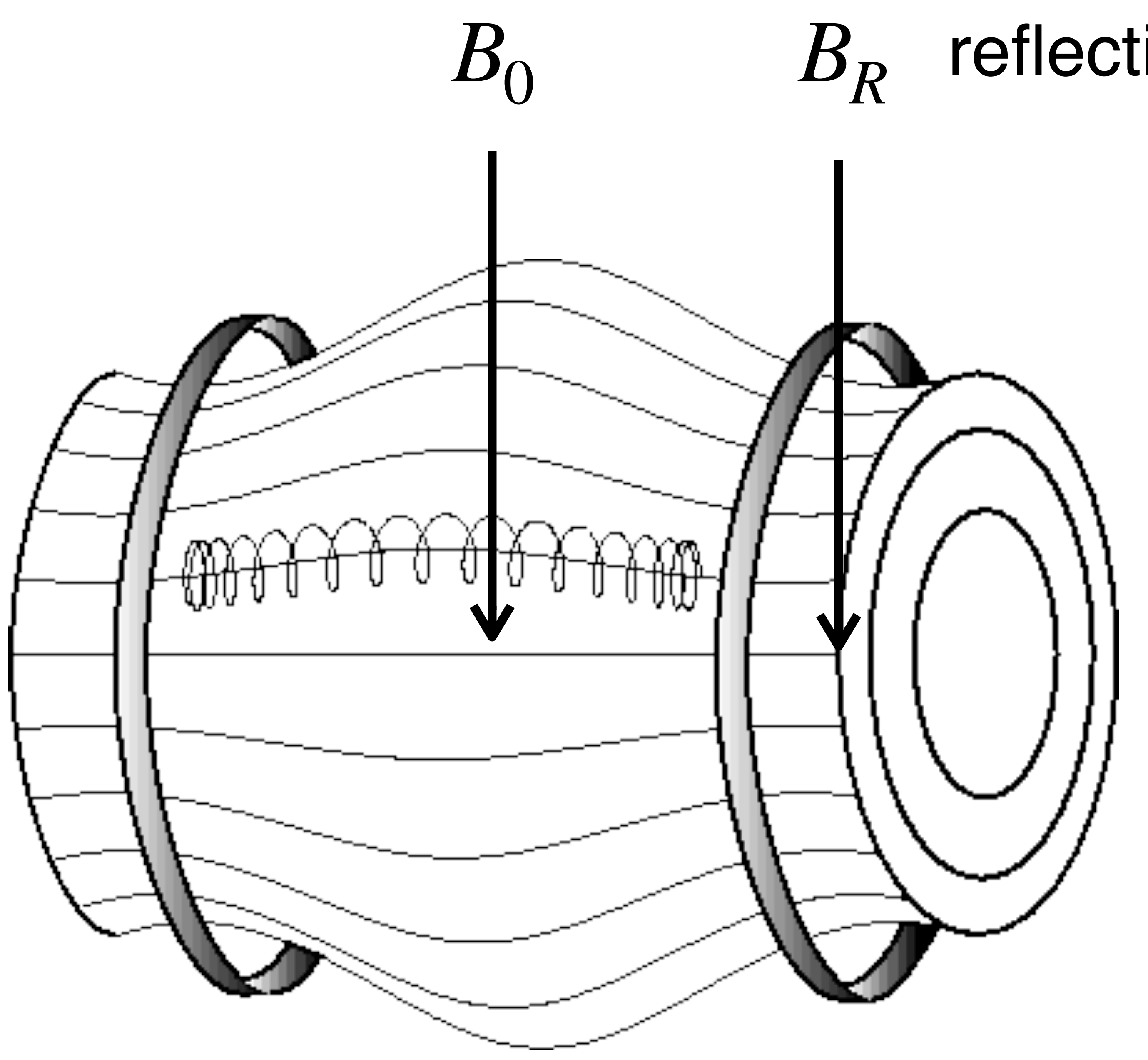
$$\tan(\theta) = \frac{|\mathbf{v}_\perp|}{|v_\parallel|}$$

- the reflection point becomes

$$\frac{B_0}{B_R} = \sin^2(\theta)$$

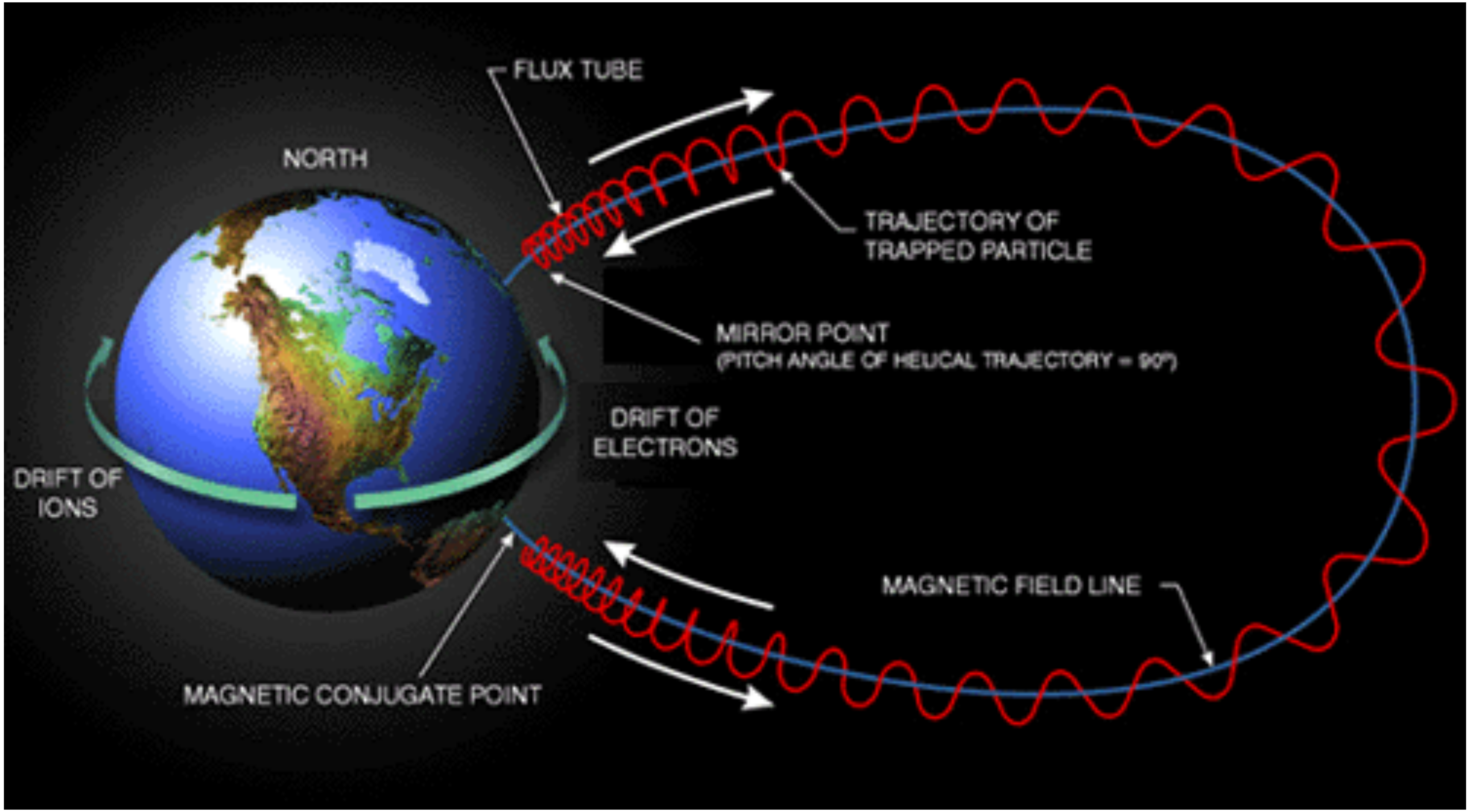
- thus, reflection occurs for angles larger than

$$\theta_c \geq \text{asin}\left[\left(\frac{B_0}{B_{\max}}\right)^{1/2}\right]$$

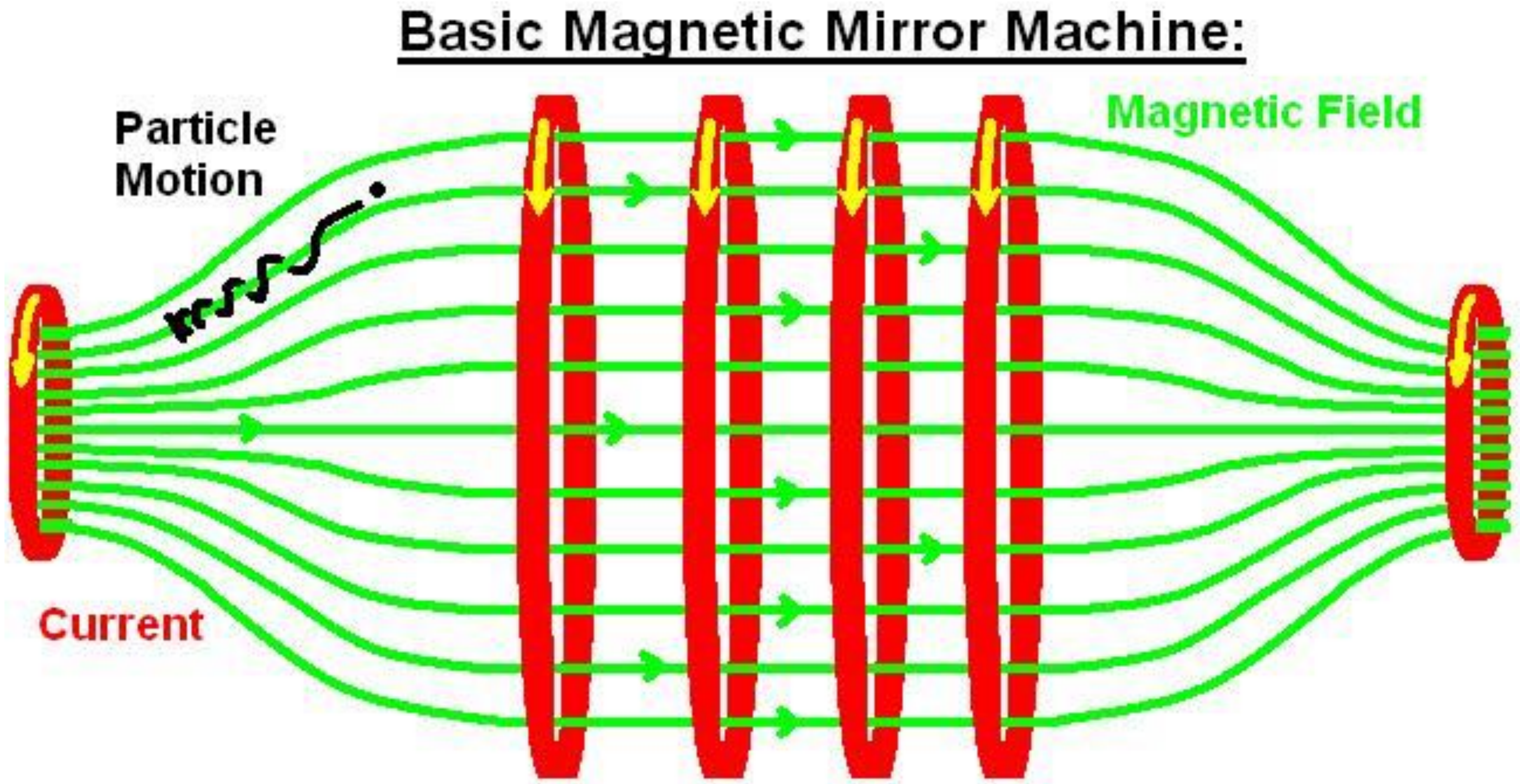


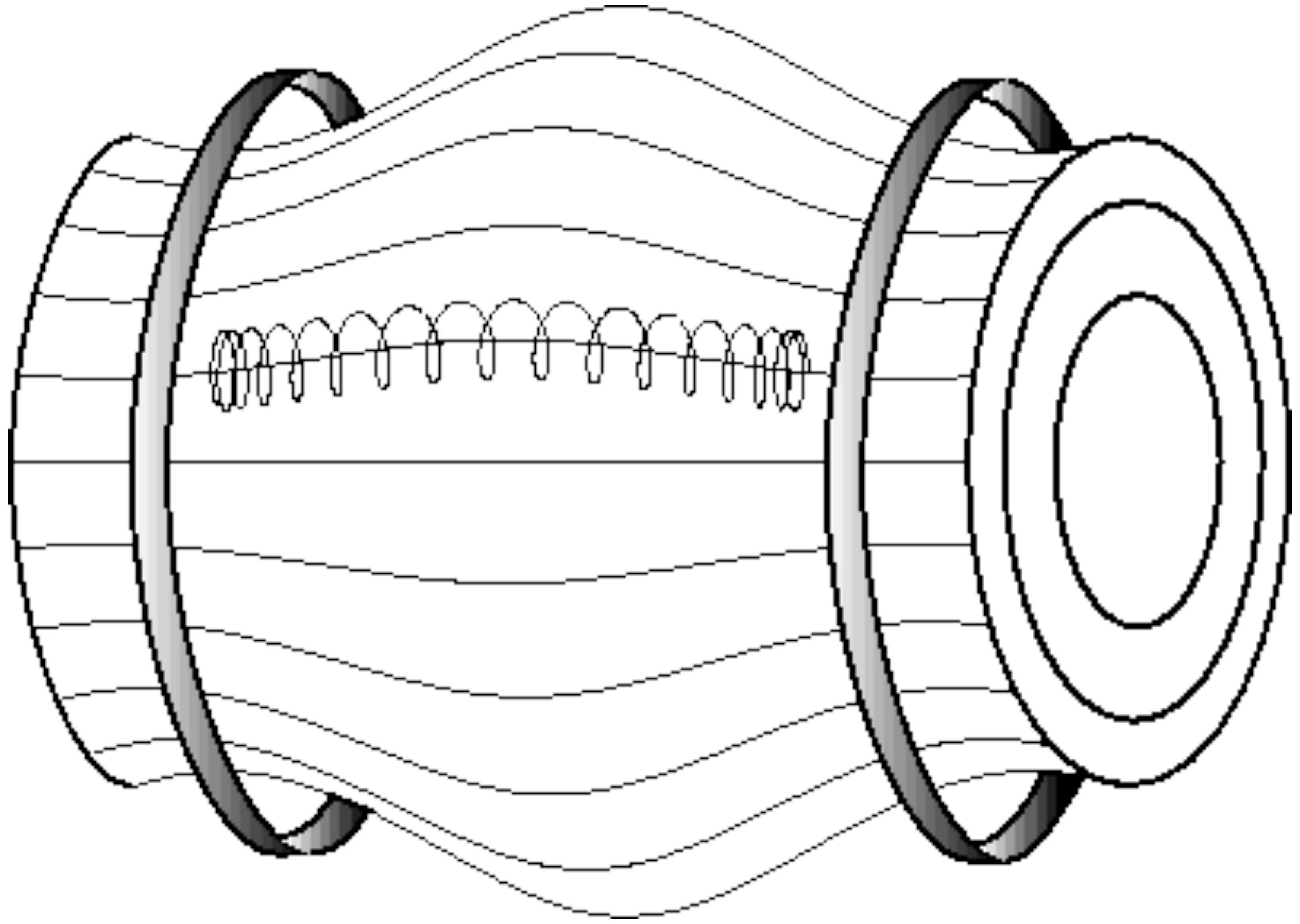
- if the mirror is moving particles can accelerate and gain energy by a process called Fermi acceleration

magnetic mirror on earth



magnetic mirror machine





- designed to confine the plasma in fusion devices
- Edward Teller showed, however, that this configuration is inherently unstable
- motivated the design of different magnetic field configurations to confine plasma in fusion devices

non-uniform B fields - curvature of the lines of force

- equation for the lines of force (we can consider only one of the terms by suitably shifting the axis)

$$\frac{\mathrm{d}x}{\mathrm{d}z} = \frac{B_x}{B_z} = \frac{1}{B_0}\left(\frac{\partial B_x}{\partial z}\right)_0 z$$

$$\begin{bmatrix} \frac{\partial B_x}{\partial x} & \frac{\partial B_x}{\partial y} & \frac{\partial B_x}{\partial z} \\ \frac{\partial B_y}{\partial x} & \frac{\partial B_y}{\partial y} & \frac{\partial B_y}{\partial z} \\ \frac{\partial B_z}{\partial x} & \frac{\partial B_z}{\partial y} & \frac{\partial B_z}{\partial z} \end{bmatrix}$$

- or

$$x = x_0 + \frac{z^2}{2B_0}\left(\frac{\partial B_x}{\partial z}\right)_0$$

- for small z the lines of force are a segment of a circle (see image next slide)

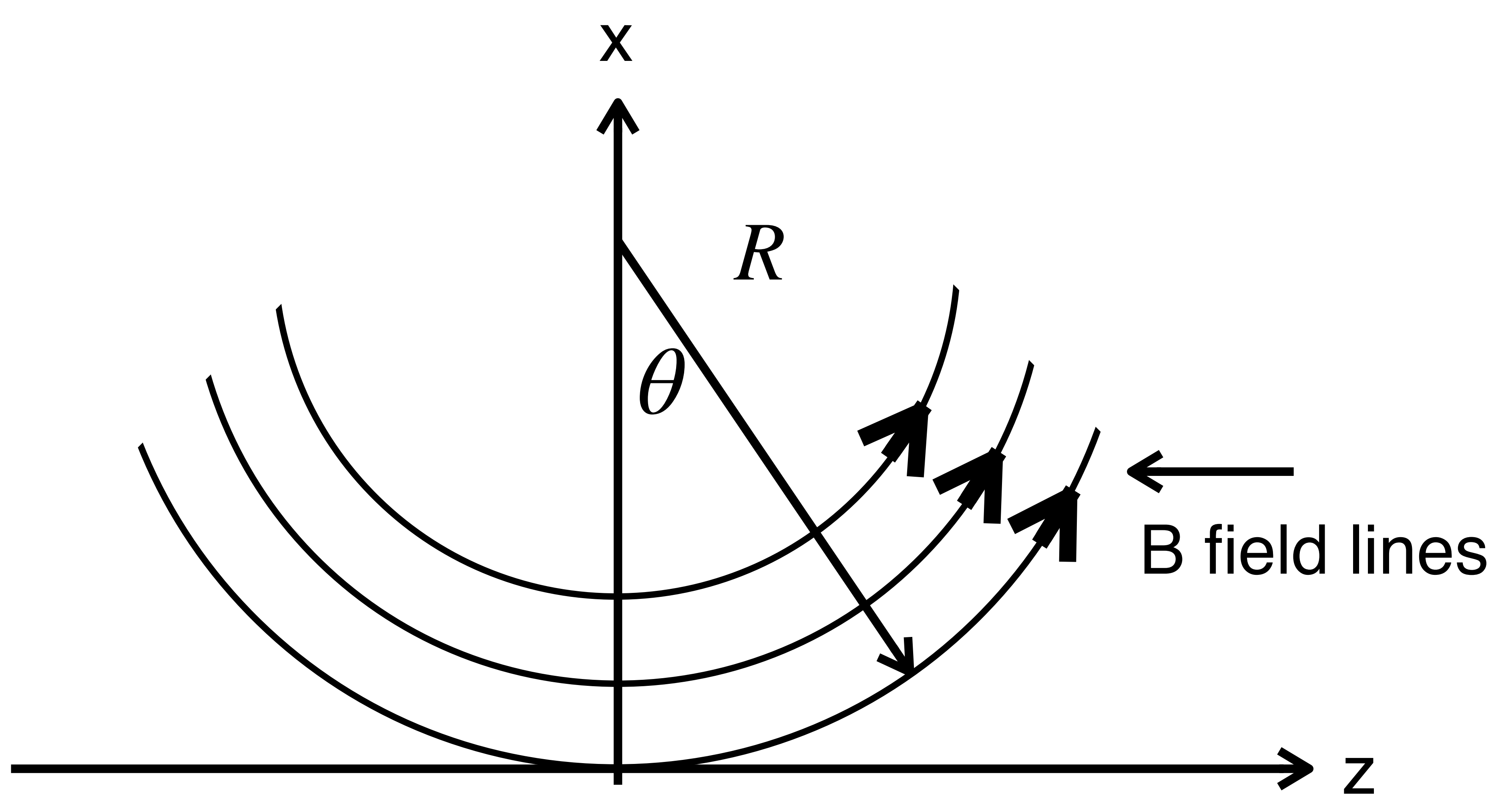
Dawson, pp 123-127

non-uniform B field - curvature of lines of force

- local approximate curved B field line by a small segment of a circle.
- assume that magnitude of B field is constant (B_0), but the direction varies.
- we can thus write (R is radius of curvature):

$$\frac{B_x}{B_0} = \tan(\theta) \simeq \theta \simeq \frac{z}{R}$$

- for small z the magnetic field is:



$$B_x = z\left(\frac{\partial B_x}{\partial z}\right)$$

radius of curvature is

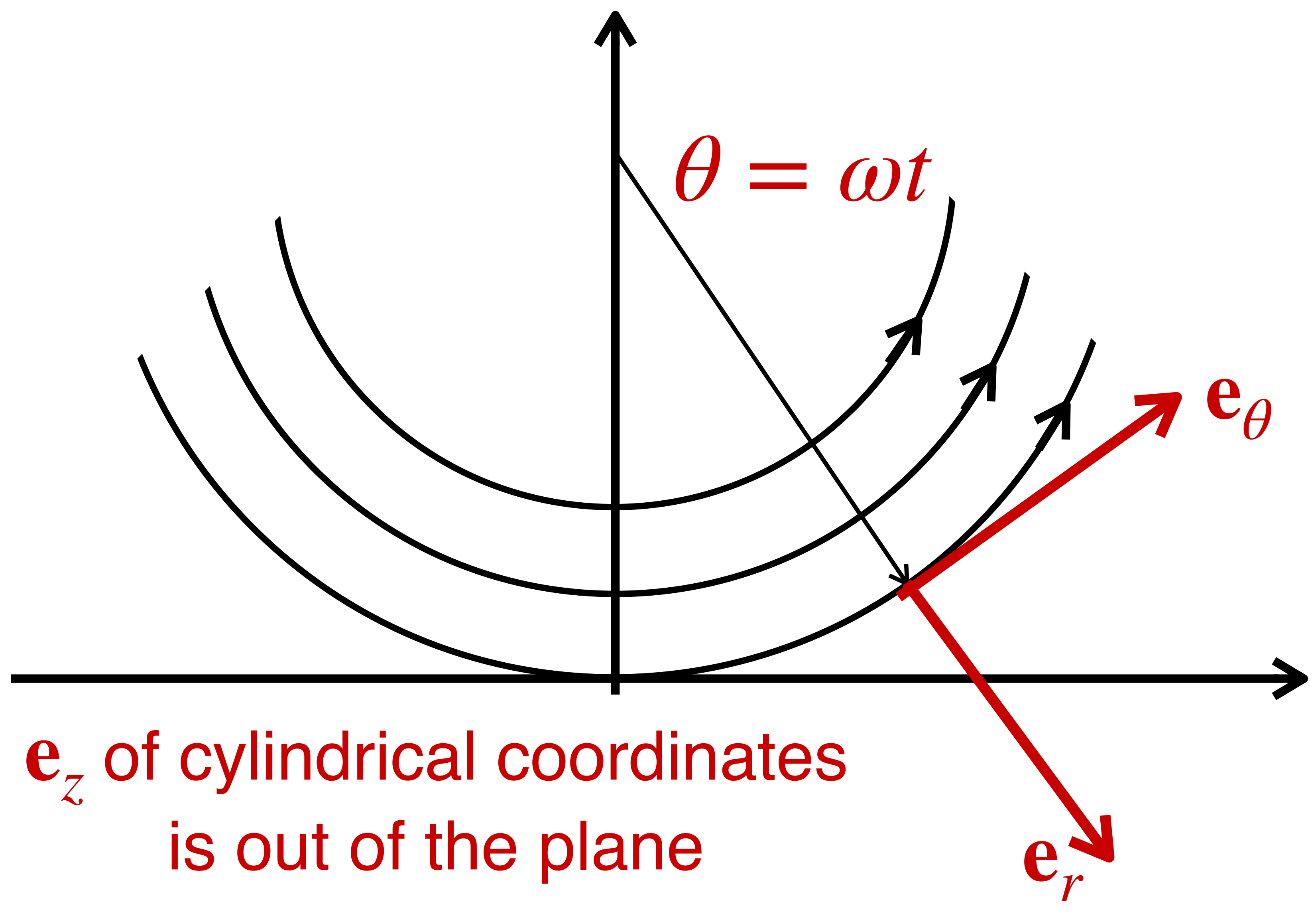
$$R = \frac{B_0}{\left(\frac{\partial B_x}{\partial z}\right)_0}$$

Dawson, pp 123-127

non-uniform B field - curvature of lines of force

- equation of motion in cylindrical coordinates

$$m\left(\mathrm{e}_r\left[\frac{\mathrm{d}v_r}{\mathrm{d}t}-\frac{v_\theta^2}{r}\right]+\mathbf{e}_\theta\left[\frac{\mathrm{d}v_\theta}{\mathrm{d}t}+\frac{v_\theta v_r}{r}+\right]+\mathbf{e}_z\frac{\mathrm{d}v_z}{\mathrm{d}t}\right)=qB_0\left(-\mathbf{e}_r v_z+\mathbf{e}_z v_r\right)$$



- the component along $\mathbf{e}_\theta$ gives conservation of angular momentum. gives rise to small variations of v_θ during cyclotron motion which we neglect.

- the other two equations are for the gyration of a particle about an uniform B field subject to an external force with magnitude

$$F_r = \frac{mv_\theta^2}{R}$$

- there is thus a drift parallel to the lines of force: $v_z = \dfrac{mv_\theta^2}{RqB_0} = \dfrac{2W_\parallel}{RqB_0}$ or $\mathbf{v}_d = \dfrac{1}{q}\dfrac{\mathbf{F}_r\times\mathbf{B}}{\mathbf{B}^2} = \dfrac{mv_\parallel^2}{q\mathbf{B}^2}\dfrac{\mathbf{e}_r\times\mathbf{B}}{R}$

Dawson, pp 123-127

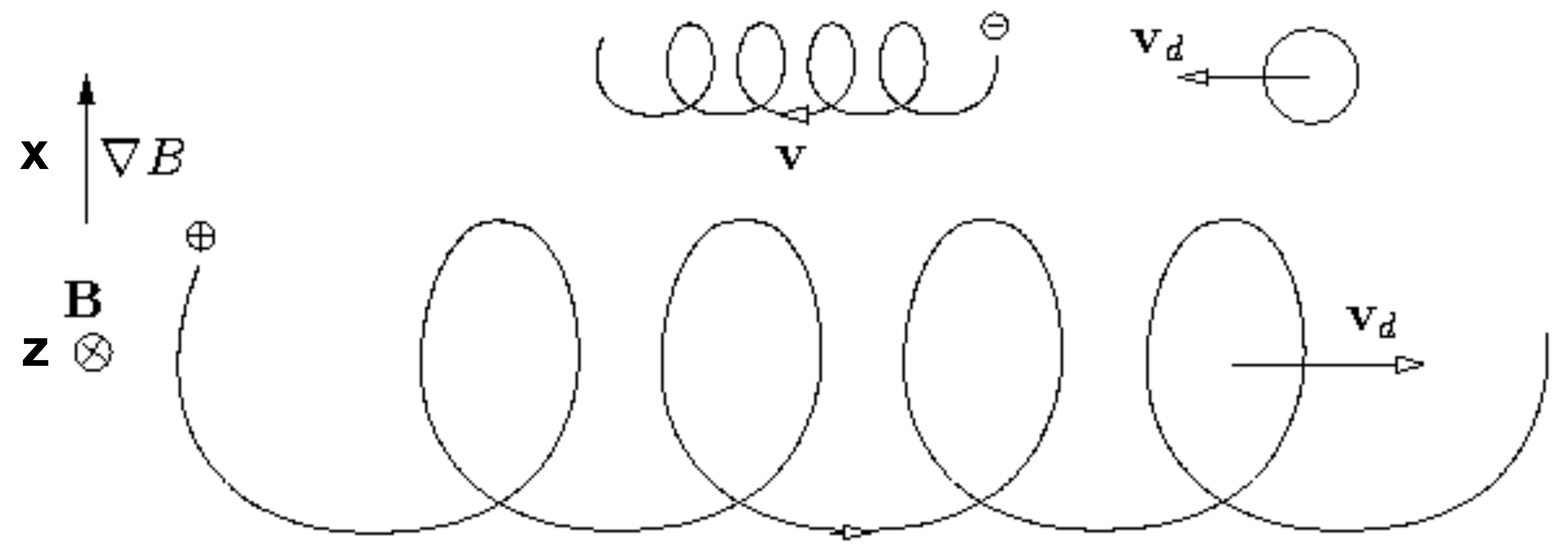
non-uniform B fields - $\nabla \mathbf{B}$ drift

- these terms only mean that the strength of the B field varies in the (x,y) plane
- by choosing the coordinate system, we can consider just one of the terms
- assume that $\partial B_z / \partial y = 0$

$$\begin{bmatrix} \frac{\partial B_x}{\partial x} & \frac{\partial B_x}{\partial y} & \frac{\partial B_x}{\partial z} \\ \frac{\partial B_y}{\partial x} & \frac{\partial B_y}{\partial y} & \frac{\partial B_y}{\partial z} \\ \frac{\partial B_z}{\partial x} & \frac{\partial B_z}{\partial y} & \frac{\partial B_z}{\partial z} \end{bmatrix}$$

Dawson, pp 127-129

non-uniform B field - $\nabla \mathbf{B}$ drift



physical picture

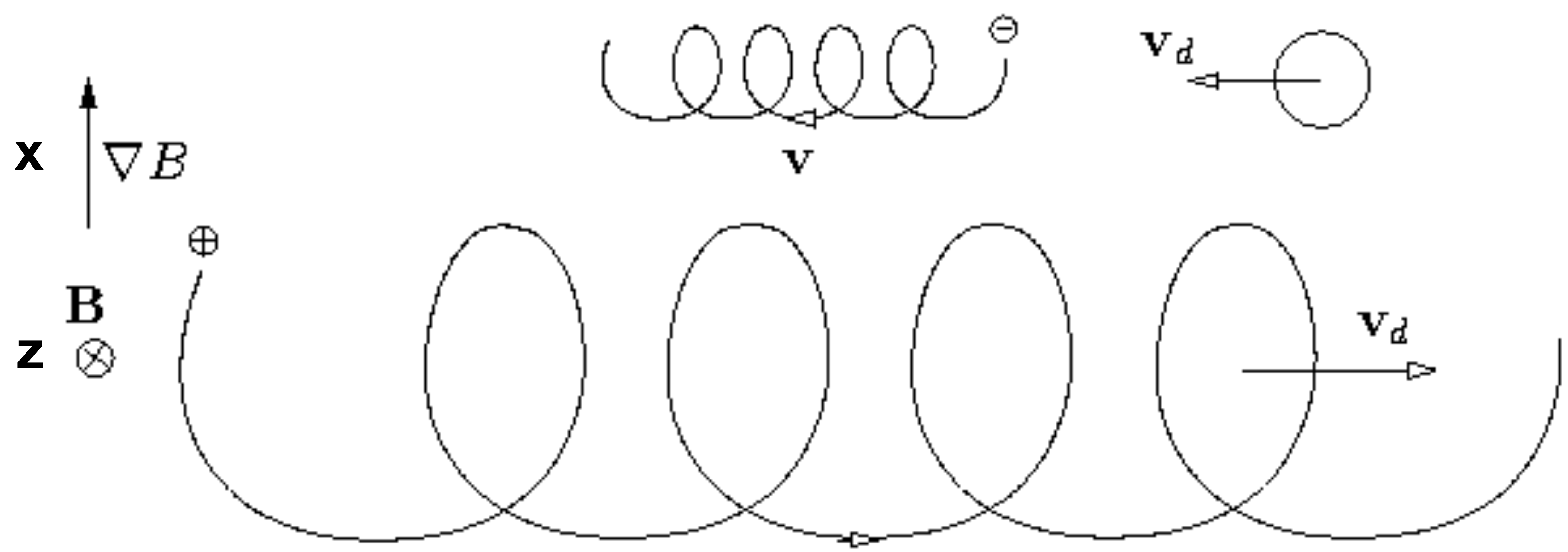
- charged particle turning in a non-uniform magnetic field experiences a drift similar to the ExB

$$r_c = \frac{m|\mathbf{v}_\perp|}{|q|\mathbf{B}}$$

- Larmor radius is larger in the part of the trajectory where B is smaller (bottom half)
- negative charges drift to the left and positive charges to the right

Dawson, pp 127-129

non-uniform B field - $\nabla \mathbf{B}$ drift



derivation

- the average force along the magnetic field gradient over one cyclotron period must be zero

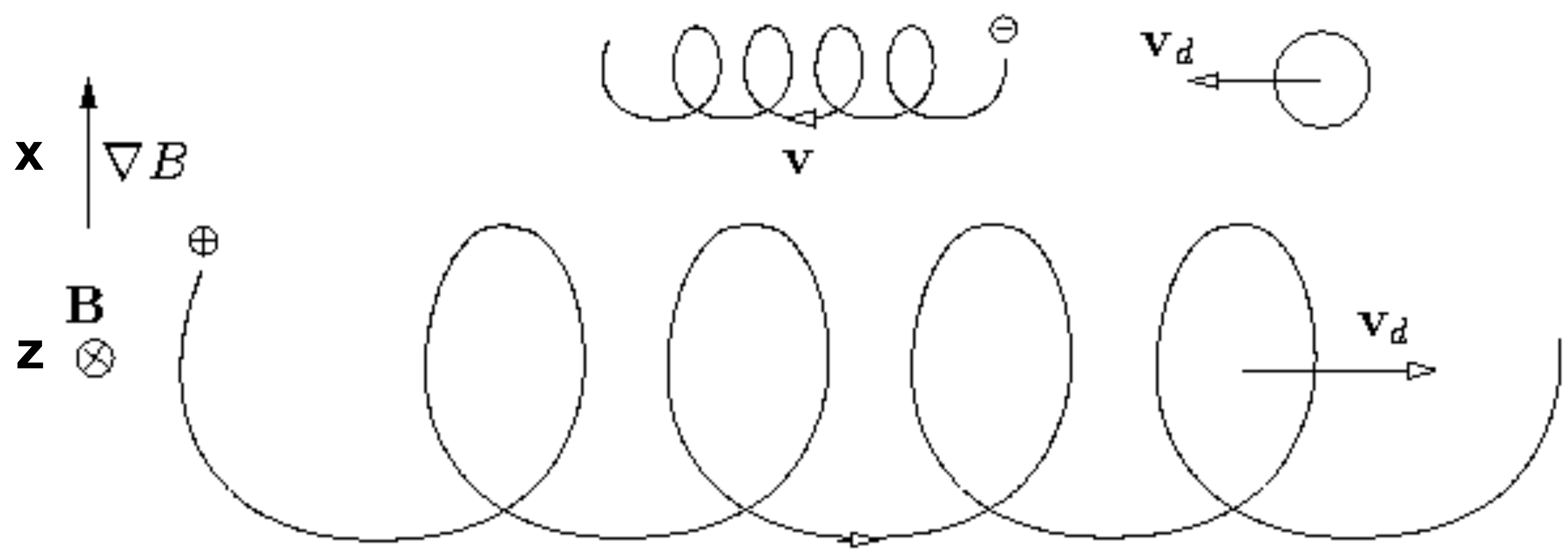
$$\int_{z1}^{z2} F_x dt = 0 \qquad F_x = q v_y B_z(x) = e v_y \left[B_0 + x \left(\frac{\partial B_z}{\partial x}_0 \right) \right]$$

- thus

$$\delta y = y_2 - y_1 = -\frac{1}{B_0} \left(\frac{\partial B_z}{\partial B_x} \right) \int_{t1}^{t2} x v_y dt$$

Dawson, pp 127-129

non-uniform B field - $\nabla \mathbf{B}$ drift



derivation

- integral over **one** period is simply $\pm\pi r_c^2$ (positive for electrons and negative for ions)
- hence

$$\delta_y = y_2 - y_1 = \pm \frac{1}{B_0}\left(\frac{\partial B_z}{\partial x}\right)\pi r_c^2$$

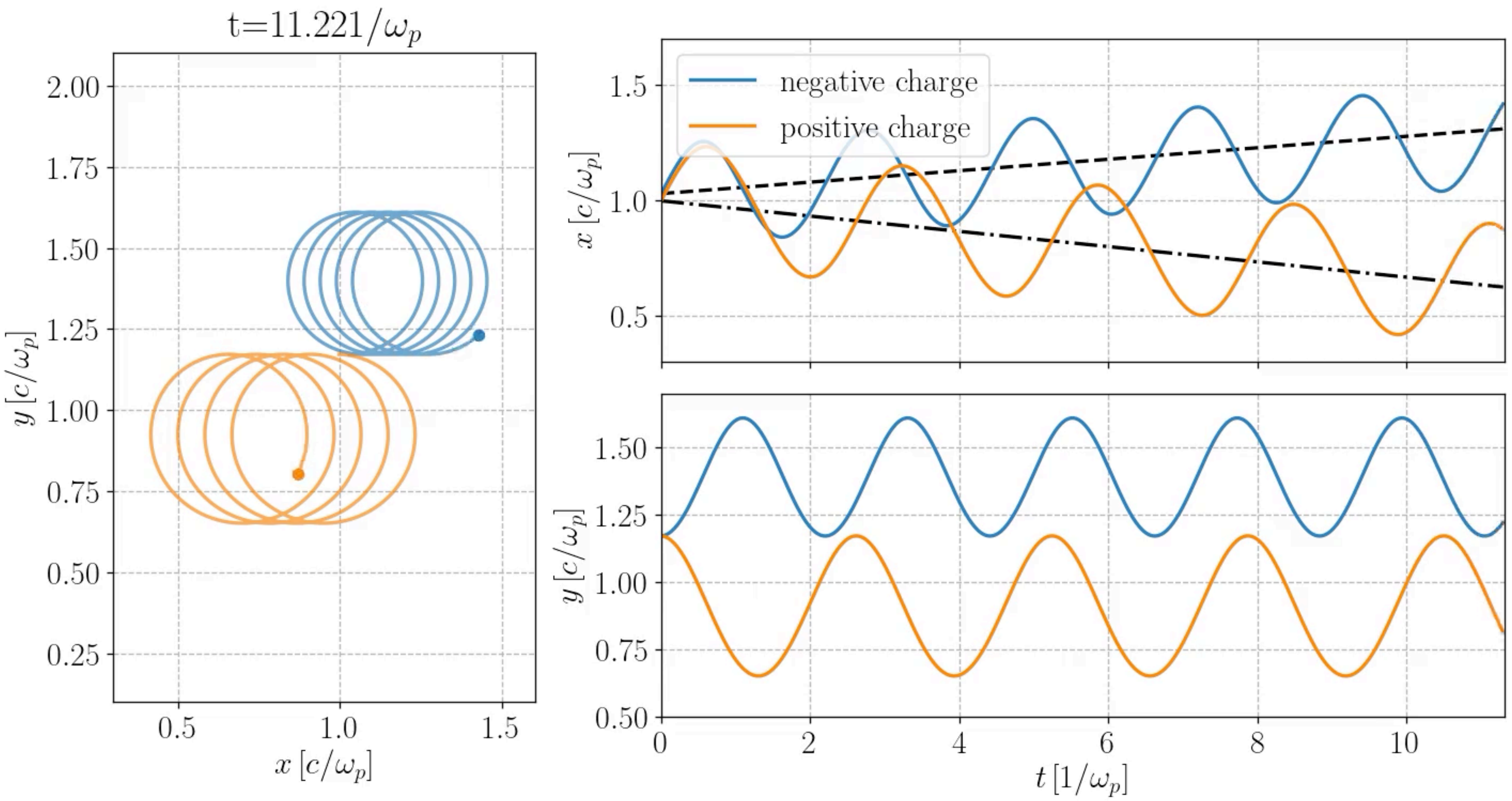
- dividing δy by the cyclotron period $2\pi/\omega_c$:

$$v_y = \frac{1}{q}\frac{m\mathbf{v}_\perp^2}{2B_0^2}\frac{\partial B_z}{\partial x} \quad \Longrightarrow \quad v_y = \frac{q}{|q|}\frac{m\mathbf{v}_\perp^2}{2B_0^2}\frac{\mathbf{B}\times\nabla\mathbf{B}}{\mathbf{B}^2}$$

Dawson, pp 127-129

Grad B drift - simulation

B field out of the plane; B field is stronger for smaller values of y



Dawson, pp 127-129