Plasma Physics and Technology
transverse and ion acoustic waves

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How to study plasmas

- single particle motion
  - simple but powerful analysis
  - enables to investigate key waves and instabilities in plasma physics

- plasma kinetic equations
  - general approach
  - can be solved using computer programs

- **fluid equations**
  - plasma waves and instabilities
  - interaction with electromagnetic waves

Transverse (electromagnetic waves) in plasmas

Maxwell's equations combined with Lorentz force

$$\nabla(\nabla\cdot\mathbf{E}) - \nabla^2\mathbf{E} + \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} + \frac{\omega_p^2}{c^2}\mathbf{E} = 0$$

- **electrostatic** (also called longitudinal) plasma waves

$$\mathbf{k}\times\mathbf{E} = 0$$

- **electromagnetic** (also called transverse) waves

$$\mathbf{k}\cdot\mathbf{E} = 0$$

- No charge separation leading space-charge effects

Initially transverse waves remain transverse for ever

- **Poisson** equation

$$\nabla\cdot\mathbf{E} = 0 \Rightarrow \frac{\partial}{\partial t}\nabla\cdot\mathbf{j} = 0$$

- If $\nabla\cdot\mathbf{j} = 0$ then:
  - $\partial n_e/\partial t = 0$
  - $\partial\nabla\cdot\mathbf{E}/\partial t = 0$

- The latter equalities hold **for ever**

- Initially transverse wave **remains** transverse

Dawson, pp. 220-222

Properties of transverse waves in plasmas

Maxwell's equations combined with Lorentz force

Wave equation in a **dispersive** medium **(plasma)**

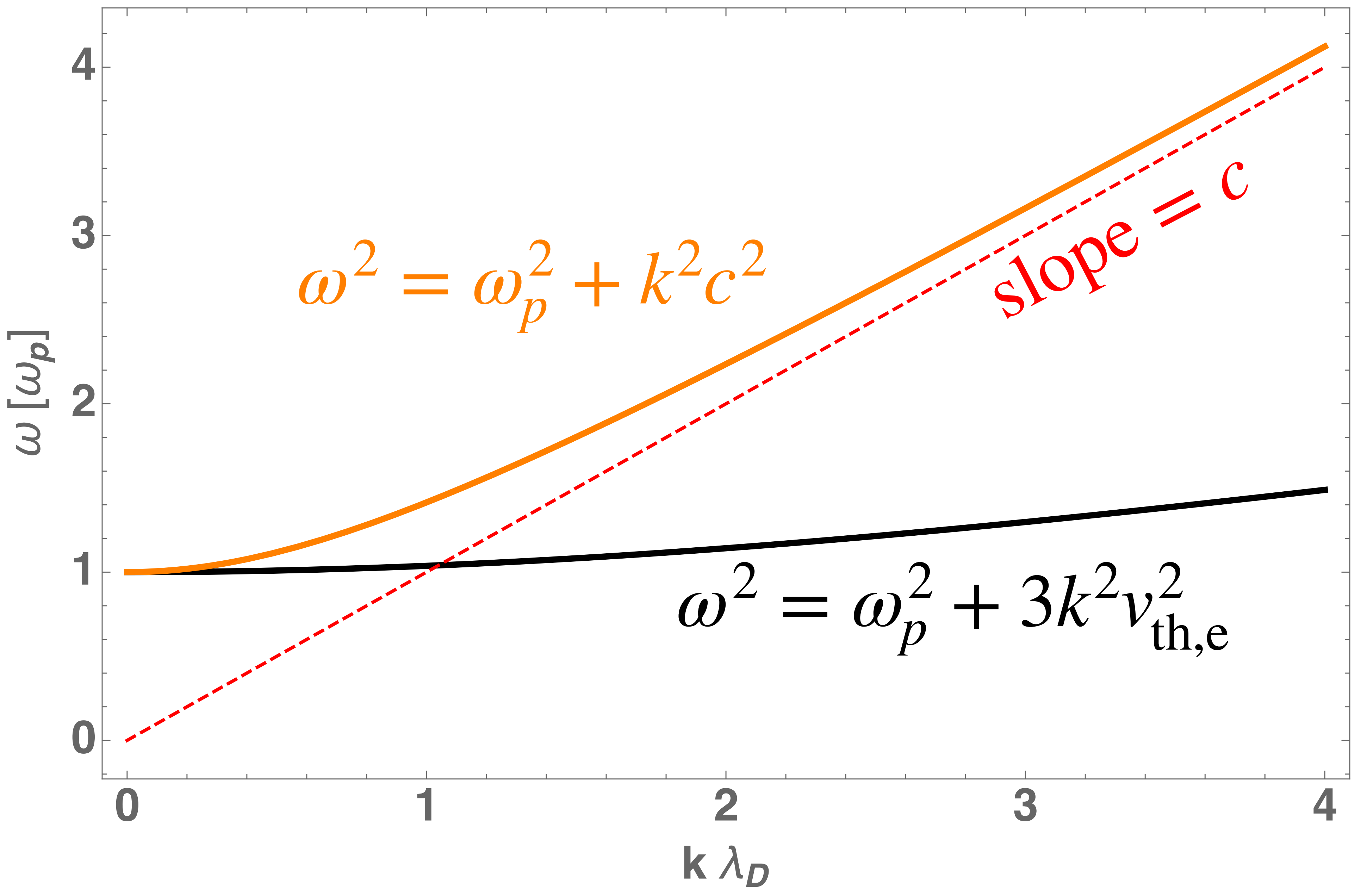
$$\nabla(\cancel{\nabla\cdot}\mathbf{E}) - \nabla^2\mathbf{E} + \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} + \frac{\omega_p^2}{c^2}\mathbf{E} = 0$$

Wave equation in a **dispersive** medium **(plasma)**

Fourier analysis

$$k^2c^2 = \omega^2 - \omega_p^2$$

Dispersion relation



Dawson, pp. 220-222

Cutt of frequency

Maxwell's equations combined with Lorentz force

- Consider that

$$\omega^2 < \omega_p^2 \Rightarrow k^2c^2 < 0$$

- k is imaginary

$$\begin{cases} \mathbf{E} \to 0 \\ \text{or} \\ \mathbf{E} \cancel{\to} \infty \end{cases}$$

Energy conservation violation

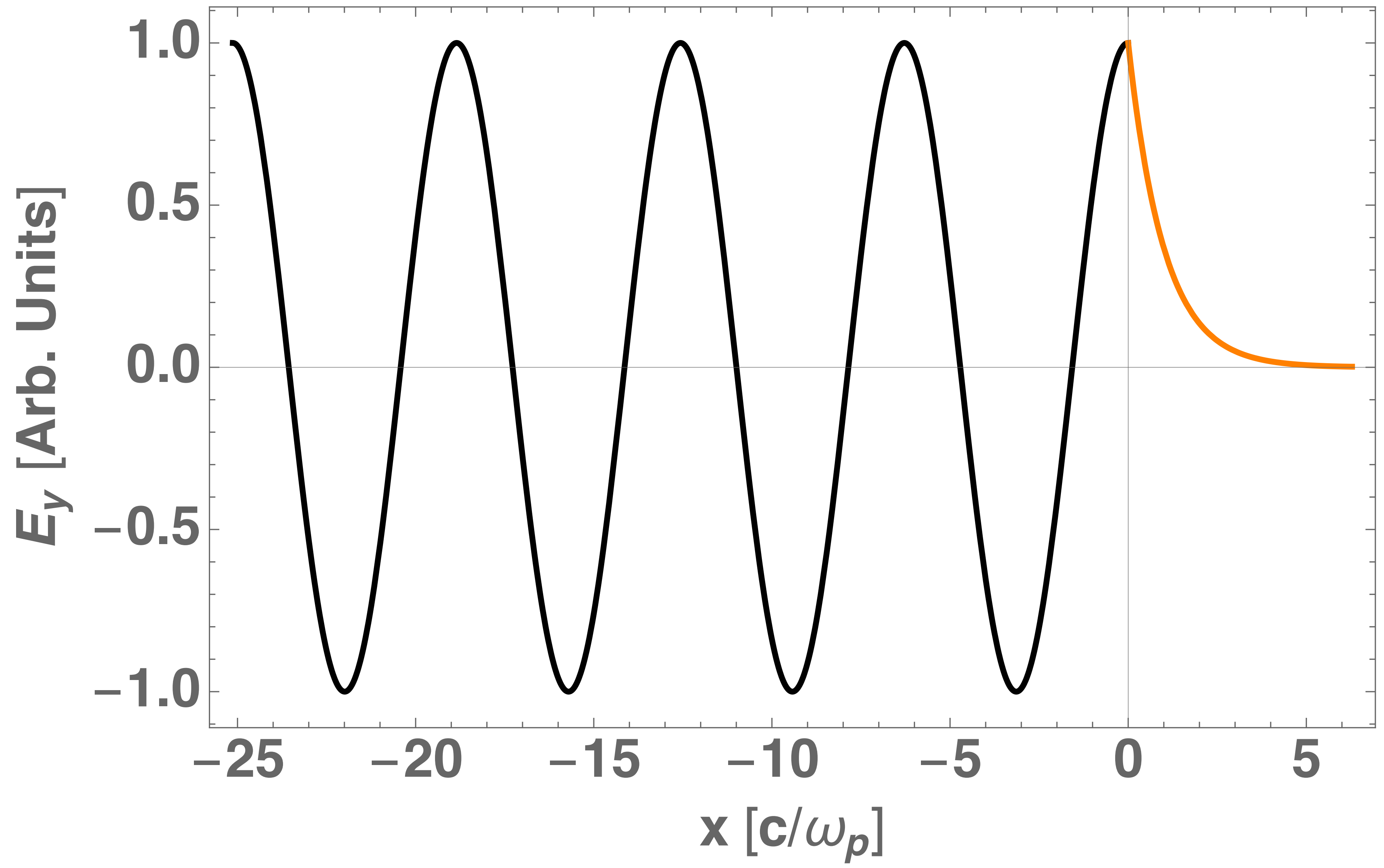
- Damped wave

$$\mathbf{E} \propto \exp(-i\mathbf{k}\cdot\mathbf{r})$$

$$kc = \sqrt{\omega_p^2 - \omega^2}$$

Cut-Off frequency

$\mathbf{E} \propto \exp(-i\mathbf{k}\cdot\mathbf{r})$ $\mathbf{E} \propto \exp(-\mathbf{k}\cdot\mathbf{r})$



Dawson, pp. 220-222

Phase velocity, group velocity, refractive index

Phase and group velocity

• **Phase** velocity

$$v_\phi^2 = \frac{\omega^2}{k^2} = \frac{c}{1 - \frac{\omega^2}{\omega_p^2}} > c$$

• **Group** velocity

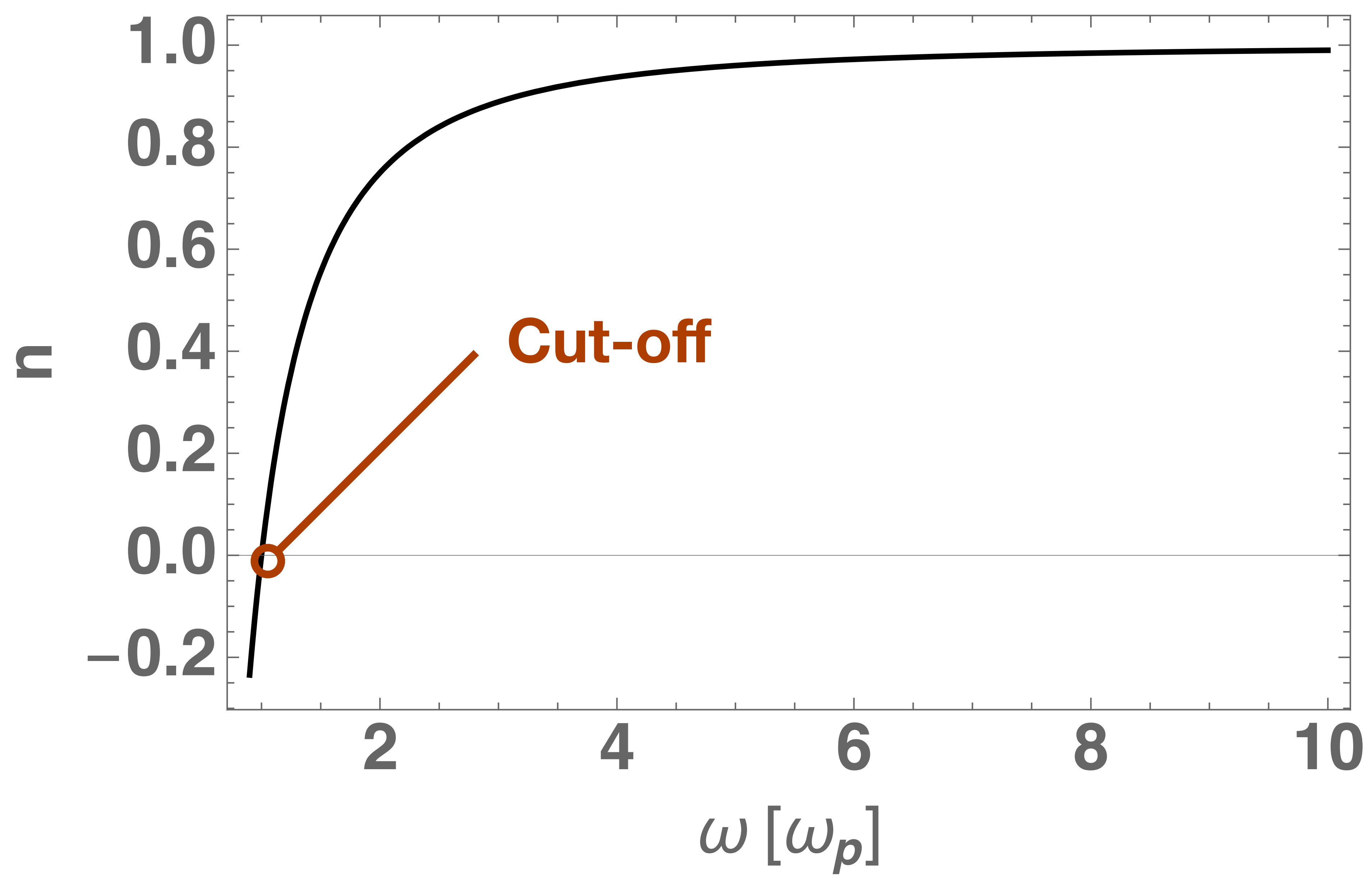
$$v_g^2 = c^2\sqrt{1 - \frac{\omega_p^2}{\omega^2}} < c$$

• **Product** between group and phase velocity

$$v_g v_\phi = c^2$$

Index of refraction

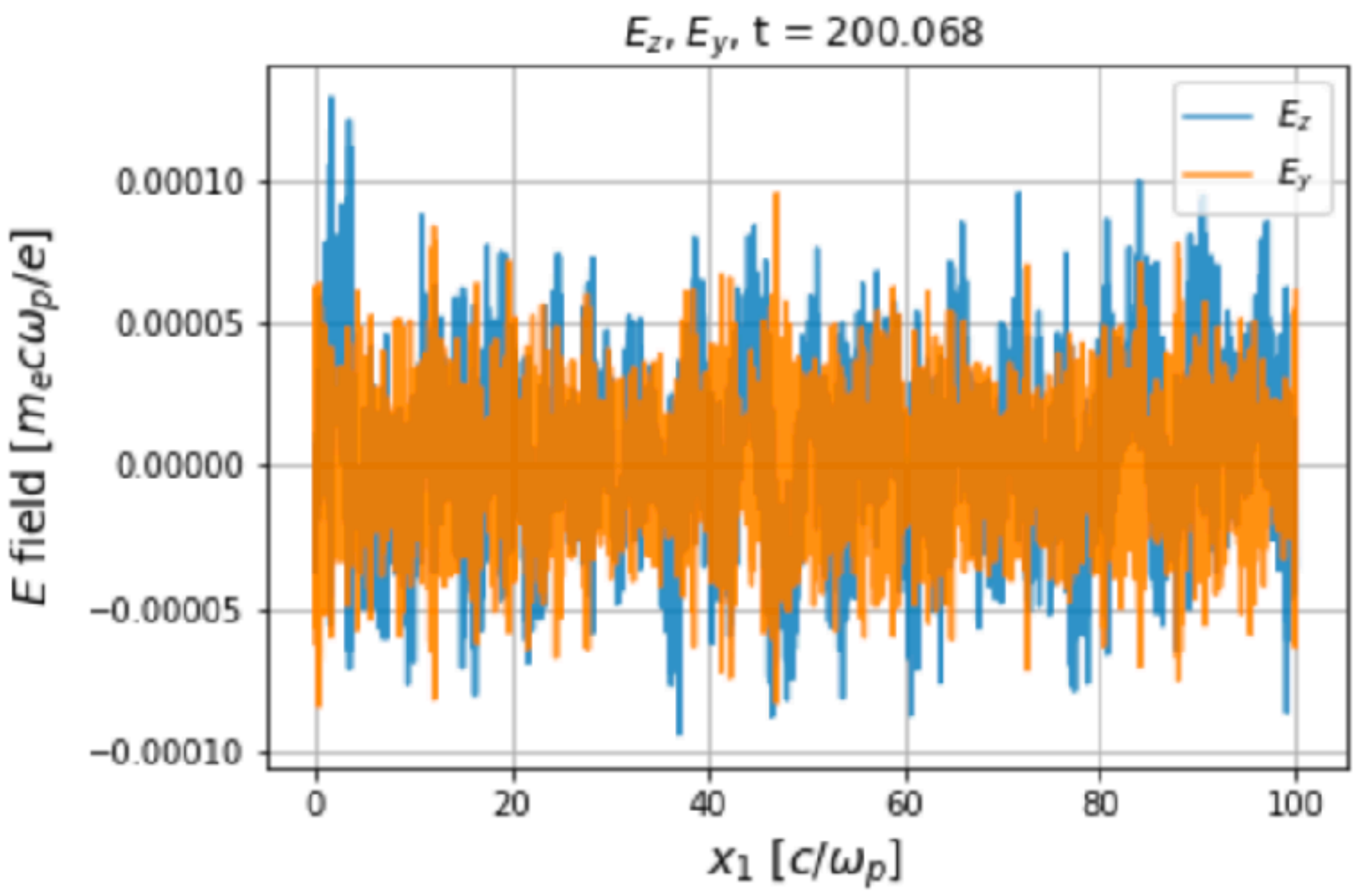
$$n = \frac{c}{v_\phi} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$



Dawson, pp. 220-222

Simulation

Transverse electric fields

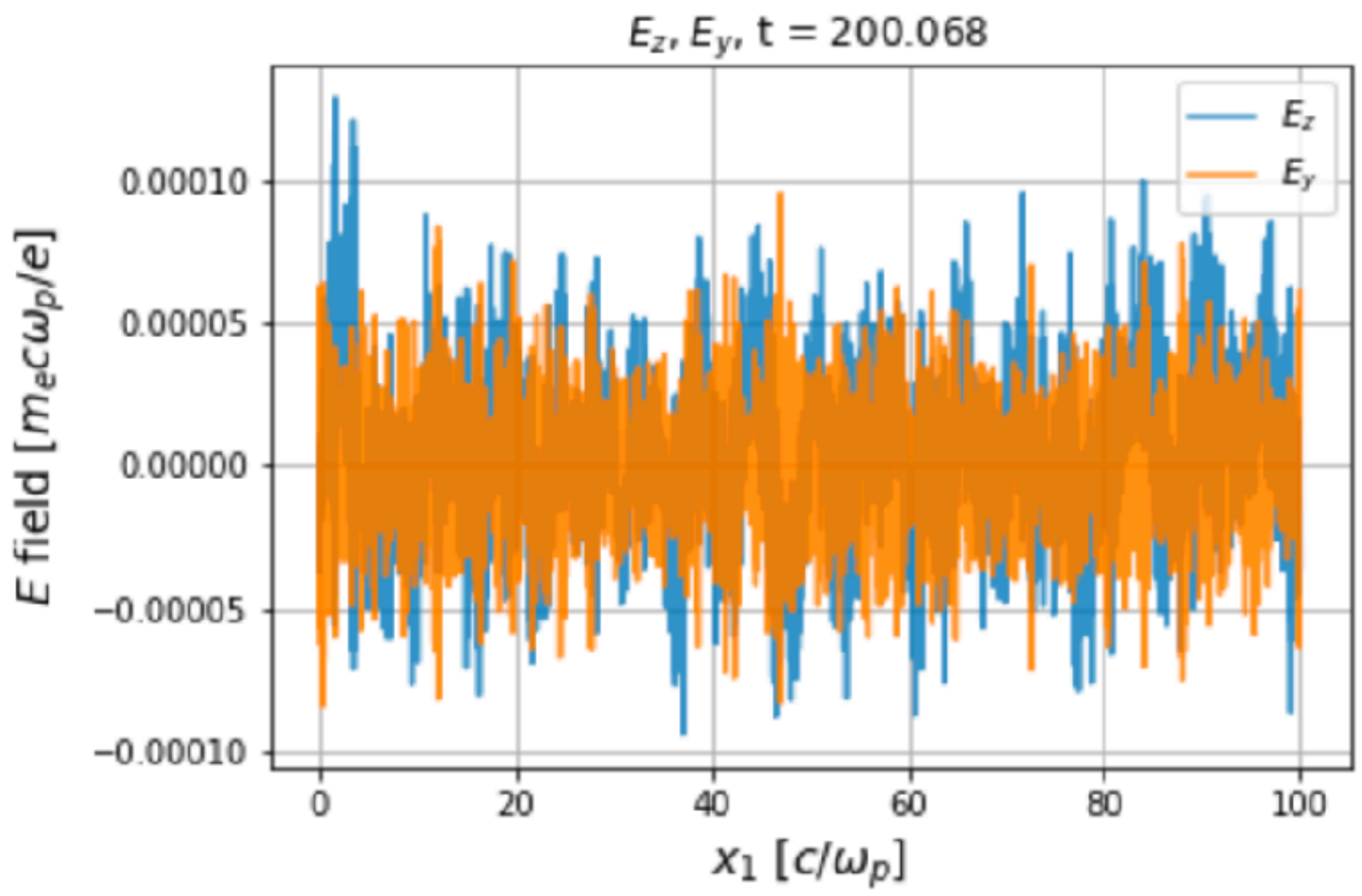


Numerical check - key simulation parameters

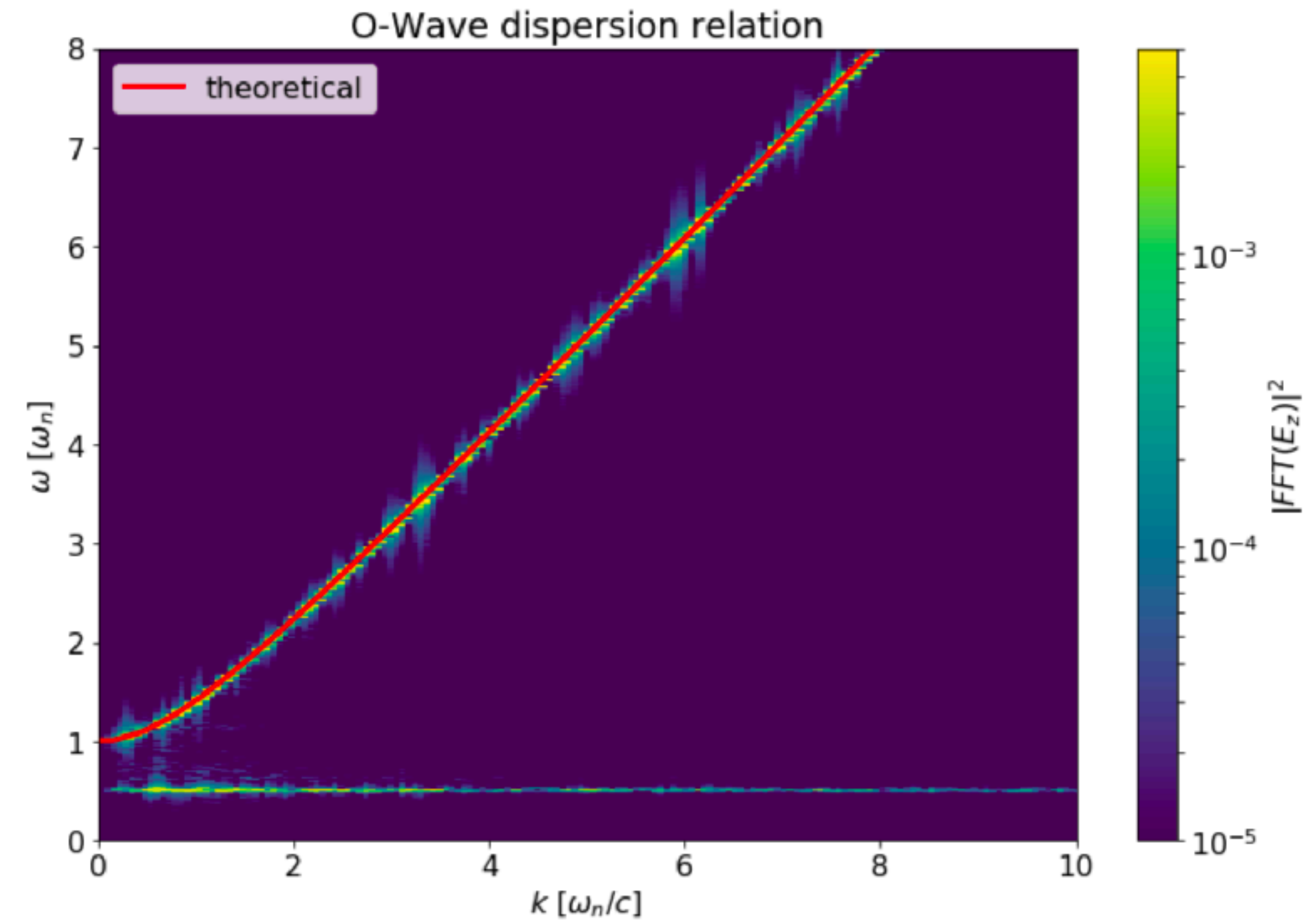
- 1D geometry
- immobile ions
- electron temperature $v_{\mathrm{th,e}} = 0.05c$ (non-relativistic!)
- box length: $100c/\omega_p$
- number of cells: 1000

Simulation

Transverse electric fields

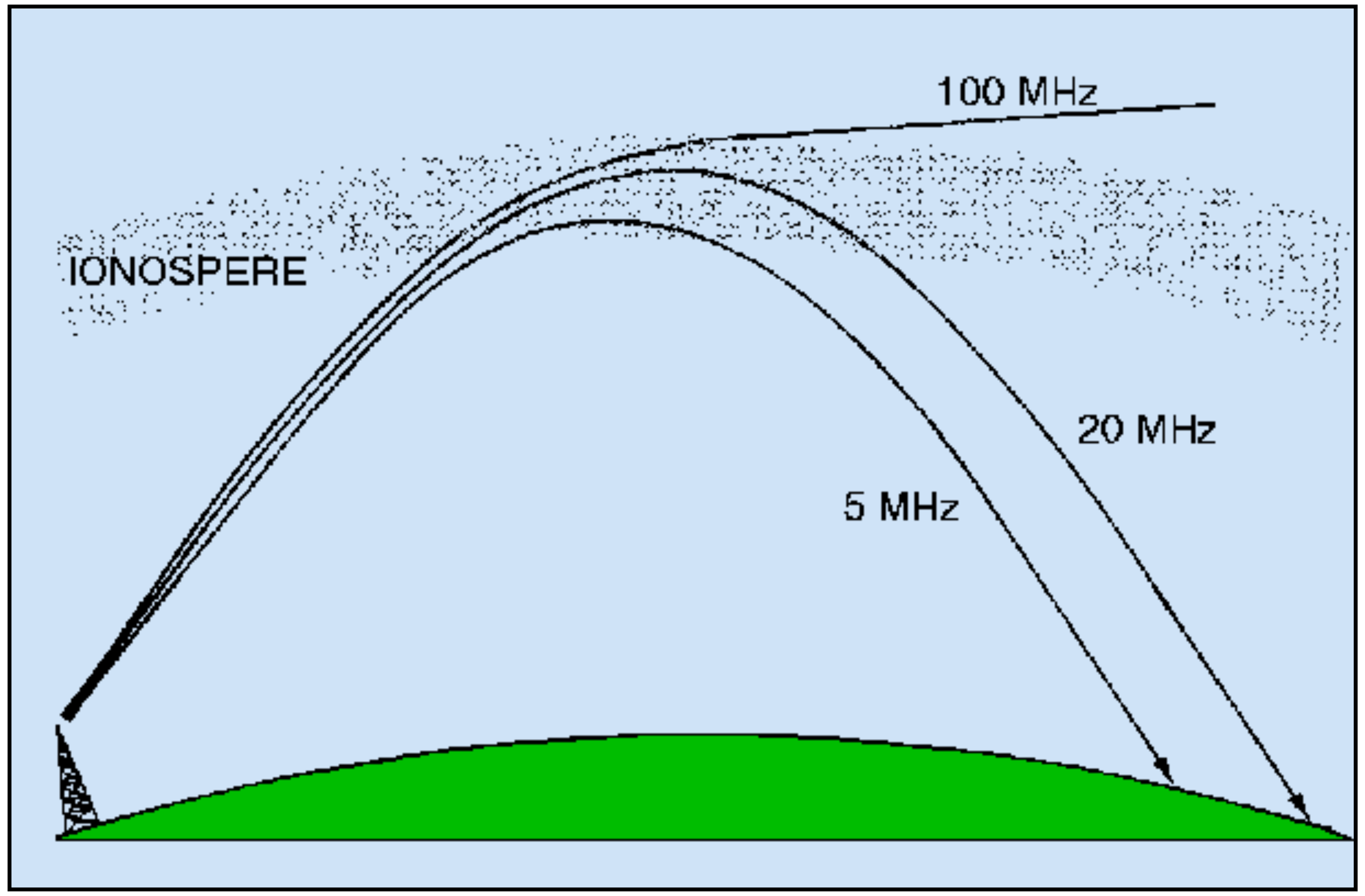


Dispersion relation



Importance of transverse waves in plasmas

Transmission of radio waves



Re-entry blackout

Spaceships in the atmospheric reentry ionize the surrounding air

- In Mars: Mars pathfinder
- In Titan: Huygens
- In Earth: space shuttle

Technology :: News Scan :: December 11, 2009 :: 33 Comments :: Email :: Print

Piercing the Plasma: Ideas to Beat the Communications Blackout of Reentry

Anticipating novel spacecraft and Mach 10 missiles, the U.S. Air Force considers new ways around an old problem

By Mark Wolverton

The frustrating communications blackout that can occur when a spacecraft reenters the atmosphere caused some tense moments in the earlier years of the space age—perhaps most memorably during the crippled *Apollo 13* mission. But the phenomenon could also affect communications with new aircraft and weapons systems being contemplated now by the U.S. Air Force, which hopes to find ways to pierce the blackout.

HOT STUFF COMING THROUGH: Computer modeling by Krishnendu Sinha of the

Ion acoustic plasma waves

Starting equations

- Electrons (n_e, v_e)
  - Continuity
  - Lorentz force
- Ions (n_i, v_i)
  - Continuity
  - Lorentz force
- Maxwell (E)
  - Poisson
- 5 coupled differential equations for 5 unknowns

Generalities about ion acoustic waves

- Electron plasma waves
  - $\omega \simeq \omega_{pe} \gg \omega_{pi}$
  - Ion motion can be safely ignored
- Ion acoustic waves
  - low frequency waves
  - $\omega \ll \omega_{pe}$
  - Ion motion plays a crucial role
- Two approaches
  - Assume quasi-neutrality (neglect Poisson)
  - Neglect electron inertia (retain Poisson)

Nicholson, pp. 138-139

Fluid equations

Electrons

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}\left(n_e v_e\right) = 0$$

$$m_e n_e \frac{\partial v_e}{\partial t} + n_e m_e v_e \frac{\partial v_e}{\partial x} = -eE_x - \frac{\partial P_e}{\partial x}$$

- Definitions for electrons
  - velocity along x: v_e
  - density: n_e
  - pressure: P_e
  - Electric field along x: E
  - Wave propagates along x
  - Mass: m_e

Ions

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}\left(n_i v_i\right) = 0$$

$$m_i n_i \frac{\partial v_i}{\partial t} + n_i m_i v_i \frac{\partial v_i}{\partial x} = eE_x - \frac{\partial P_i}{\partial x}$$

- Definitions for ions
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Nicholson, pp. 138-139

Ion acoustic waves - simpler theoretical approach

Poisson

$$\frac{\partial E}{\partial x} = \frac{e}{\epsilon_0}\left(n_i - n_e\right)$$

- Definitions Poisson
  - Elementary charge: e

Pressure

$$\frac{\partial P_{i,e}}{\partial x} = \gamma K T_{e,i} \frac{\partial n_{e,i}}{\partial x}$$

- Definitions
  - γ is a constant (depends on the details of the dynamics)
  - Electron/ion temperature: $T_{e,i}$

Ansatz - neglect Poisson

- Light electrons follow ions
  - electrons are quickly dragged by the ions
  - space charge effects are negligible
  - assume $n_e \simeq n_i \Rightarrow v_e \simeq v_i = v$
  - neglect Poisson's equation

Nicholson, pp. 138-139

Ion acoustic waves - simpler theoretical approach

Continuity + Force

Add electron and ion force equations and linearise

$$(m_e + m_i)n_0\frac{\partial v}{\partial t} = -K\left(\gamma_e T_e + \gamma_i T_i\right)\frac{\partial n_1}{\partial x}$$

$$\frac{\partial n_1}{\partial t} + n_0\frac{\partial v}{\partial x} = 0$$

Equation for acoustic (sound waves): **valid for small k**

$$\frac{\partial^2 n_1}{\partial t^2} - c_s^2\frac{\partial^2 n_1}{\partial x^2} = 0 \Leftrightarrow \omega^2 = c_s^2 k^2$$

Sound speed

$$c_s^2 = \frac{k_b\left(\gamma_e T_e + \gamma_i T_i\right)}{m_e + m_i} \simeq \frac{k_b\left(\gamma_e T_e + \gamma_i T_i\right)}{m_i}$$

$\gamma_{e,i}$?

- ions:
  - Adiabatic - no collisions 1D: $\gamma_i = 3$
  - Adiabatic - collisions 3D: $\gamma_i = 5/3$
- electrons:
  - Travel many wavelengths in a single period

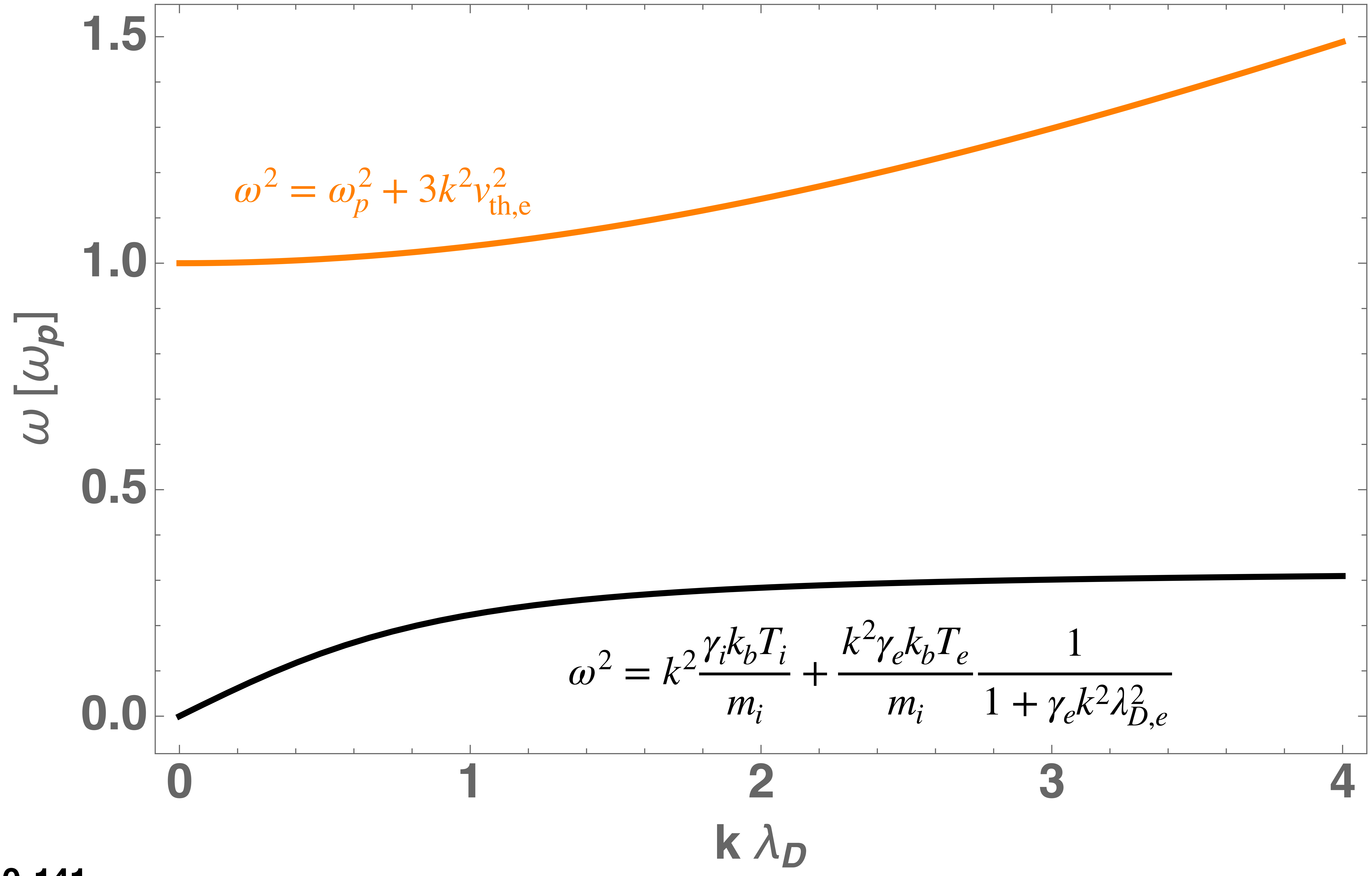
$$\frac{v_{\mathrm{th},e}}{\omega} \simeq \frac{v_{\mathrm{th},e}}{kc_s} \gg k^{-1}$$

$$\frac{v_{\mathrm{th},e}}{c_s} = \left(\frac{T_e}{m_e}\frac{m_i}{\gamma_i T_i + \gamma_e T_e}\right)^{1/2} \gg 1$$

  - Communicate over many wavelengths
  - Isothermal $\gamma_e = 1$

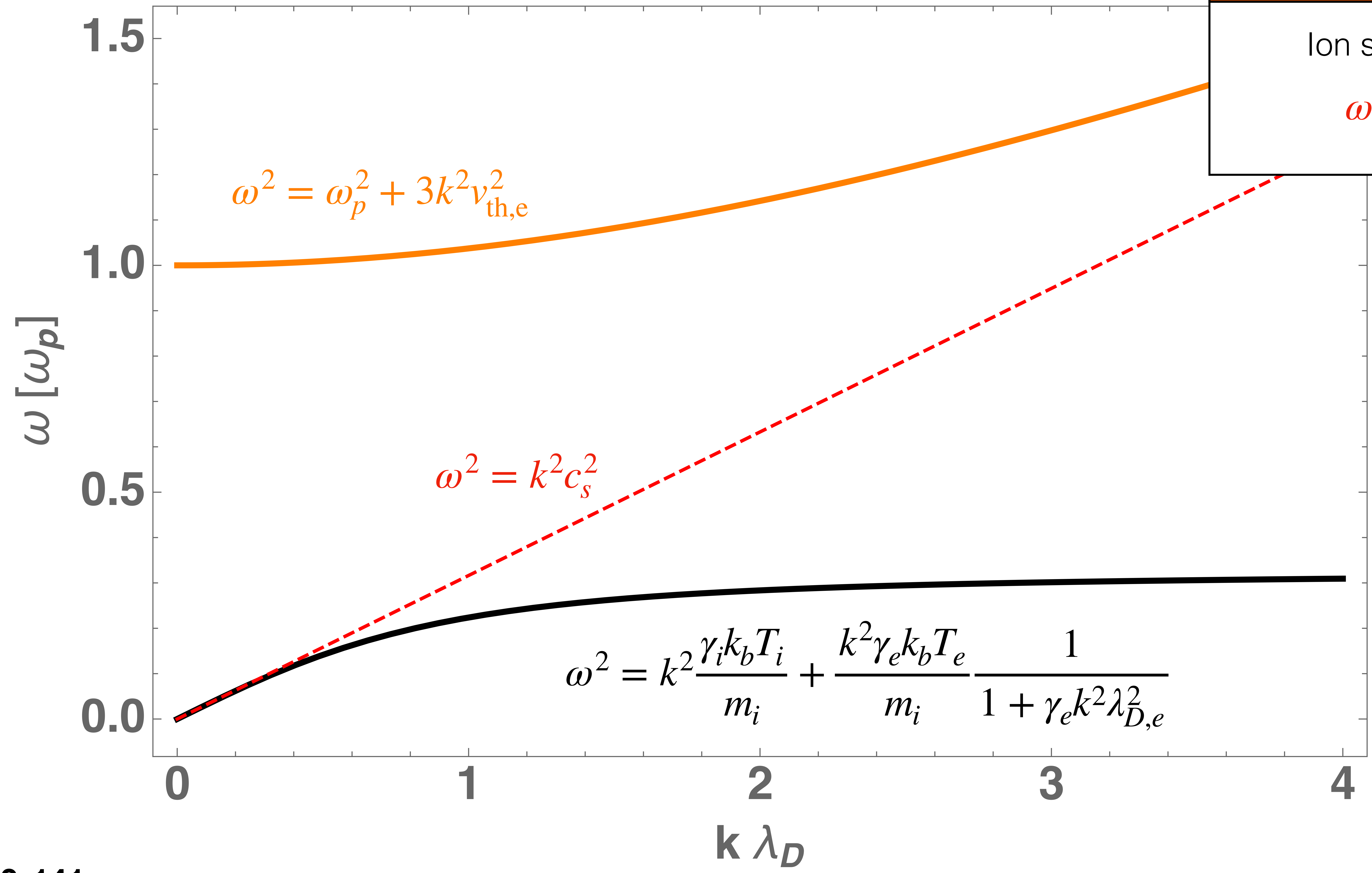
Nicholson, pp. 139-140

Ion acoustic waves - keep Poisson, neglect electrons inertia



Nicholson, pp. 140-141

Ion acoustic waves - keep Poisson, neglect electrons inertia



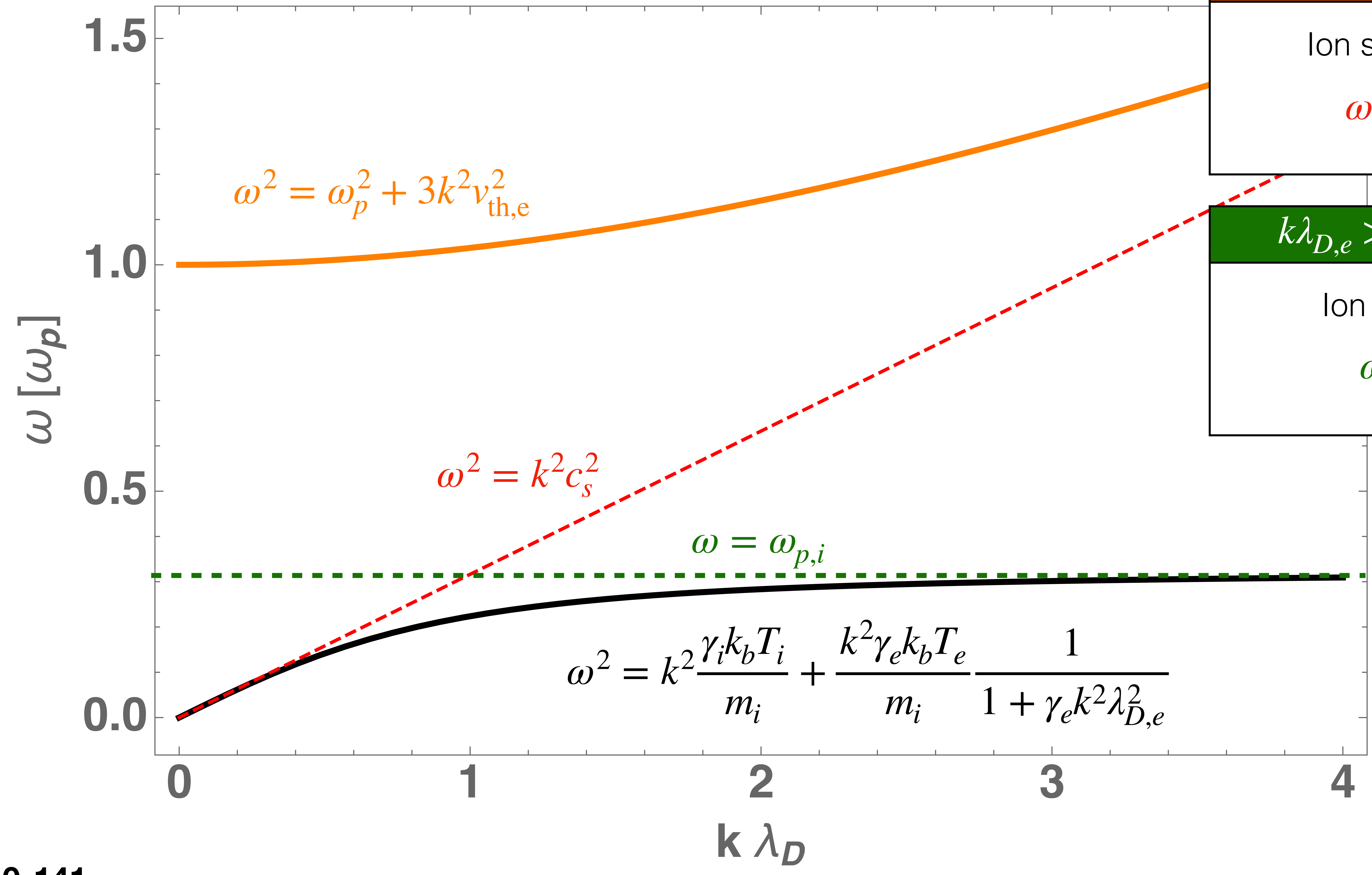
$k\lambda_{D,e} \ll 1$

Ion sound waves

$\omega^2 = k^2 c_s^2$

Nicholson, pp. 140-141

Ion acoustic waves - keep Poisson, neglect electrons inertia



$k\lambda_{D,e} \ll 1$

Ion sound waves

$$\omega^2 = k^2 c_s^2$$

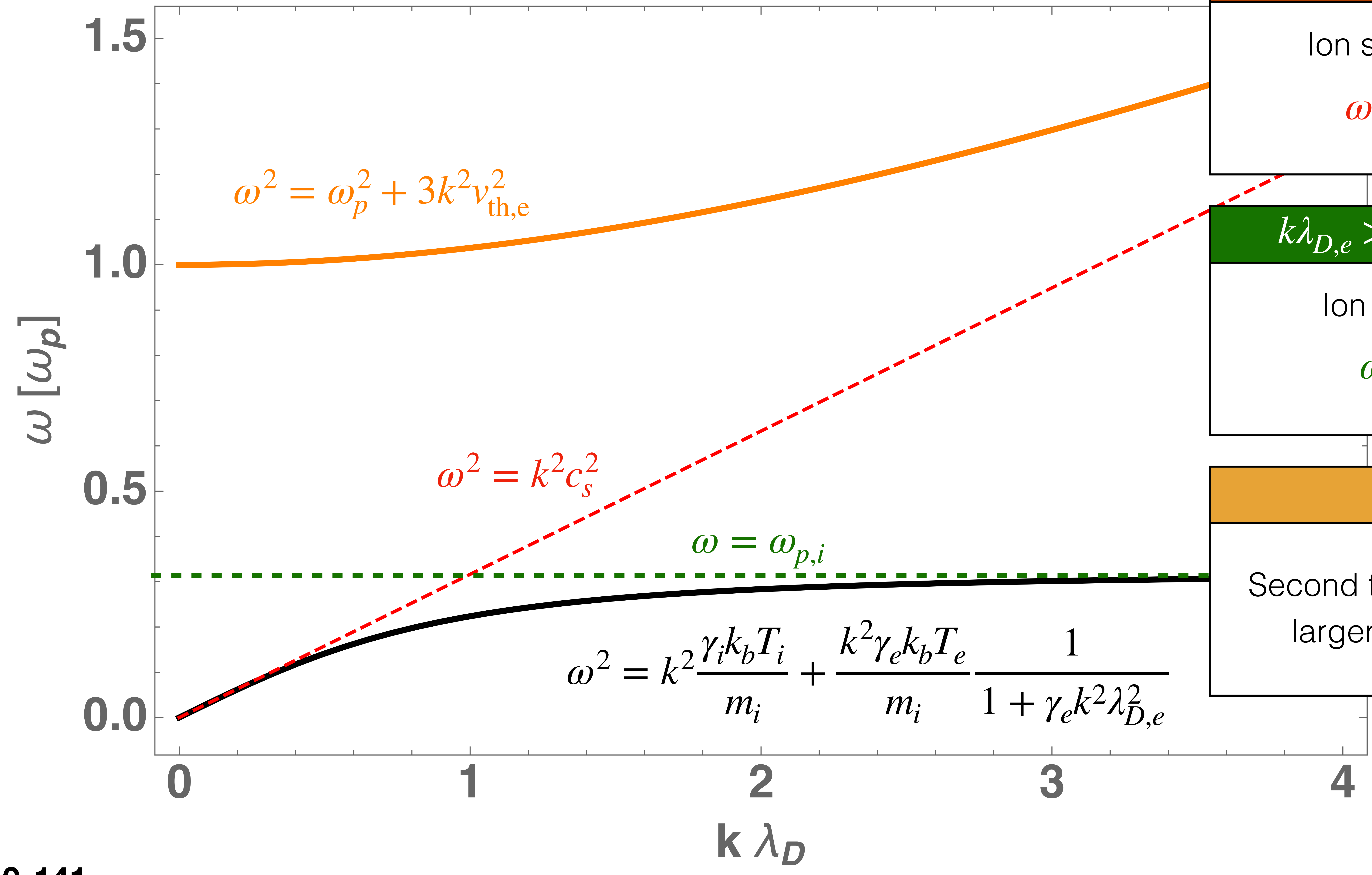
$k\lambda_{D,e} \gg 1$ & $T_i = 0$

Ion oscillations

$$\omega = \omega_{p,i}$$

Nicholson, pp. 140-141

Ion acoustic waves - keep Poisson, neglect electrons inertia



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Ion sound waves

$$\omega^2 = k^2 c_s^2$$

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Ion oscillations

$$\omega = \omega_{p,i}$$

validity

Second term on the right larger than the first

Nicholson, pp. 140-141